# Search for a rendezvous with lost target at sea 

Malika Meghjani and Gregory Dudek


#### Abstract

This paper addresses the search problem for lost floating targets on sea surface. Given the expected search region, we analyze spiral search patterns for finding the target. Specifically, we consider two search patterns: outward spiral and inward spiral. The outward spiral pattern is a greedy strategy which initializes from the mean of the search region and expands outwards to cover the entire region. In contrast, the inward spiral pattern first encapsulates the search area and then moves inwards towards the mean. We hypothesize that the inward spiral patterns give guaranteed search outcomes for the lost targets whereas the outward spiral patterns minimize the search time but not necessarily guarantee a successful outcome. We present a theoretical analysis to parametrize the search problem and we validate it using our simulator for realistic sea trials. We experimentally test the effects of varying the initial location of the lost target and the communication radius between the target and the searching agent. Our simulated results confirm our hypothesis and analytical findings.


## I. INTRODUCTION

We present a search strategy for finding lost targets at sea. An example scenario for our work is searching for lost divers or floating black boxes. We consider that the expected search region for the target is given with the target either inside the region or just about to drift outside the region. In such a case, we propose to use a spiral search strategy to cover the given area of interest. Spiral patterns are known to be natural search strategies by both animals and humans [1]. In addition, the optimal control strategy for search was also proven to be a spiral pattern by Bourgault et al.in [2].

## II. GUARANTEED SEARCH

We consider the problem of finding a point-object with a moving searcher, for example a robot boat searching for a small lost target object at sea. We formalize this by assuming the target is a point and that the robot is a disk of radius b , and that the target is known to be confined initially somewhere within a disk of radius $r$ and to be moving with constant velocity. Alternatively, we can consider the robot to also be a point but to have a 'communication radius' b (what defines a disc around the robot) so that if the target is within the disc the search terminates successfully.

Consider a two dimensional search region which is defined by a probability distribution function with value zero beyond radius $r$. A robot with maximum speed $s_{r}$ and communication radius $b$ is required to search this given area. The robot performs circular patterns to cover the search area by clearing a channel of radius $b$ per circular round. The total number of

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Fig. 1: Simulated spiral search pattern on Microsoft Bing Maps (c). The red dots indicate the waypoints of an ASV and the yellow dots represent the drifter waypoints.
circular rounds that the robot needs to complete for clearing the entire search region is given by

$$
\begin{equation*}
n_{s}=\left\lceil\frac{r}{b}\right\rceil \tag{1}
\end{equation*}
$$

The time taken to clear one circular round with radius $r$, is

$$
\begin{equation*}
\tau=\frac{2 \pi r}{s_{r}} \tag{2}
\end{equation*}
$$

The total time taken by the robot to clear the complete search area is less than or equal to the product of the total number of circular rounds and the time taken to clear each round.

$$
\begin{equation*}
\tau_{t o t} \leq n_{s} \tau \tag{3}
\end{equation*}
$$

If the robot performs a circular search pattern with diminishing radius $b$ for each circular round. Then, the total time is calculated as,

$$
\begin{equation*}
\tau_{t o t}=\frac{2 \pi n_{s}}{s_{r}} \sum_{i=0}^{\left(n_{s}-1\right)}(r-i b) \tag{4}
\end{equation*}
$$

## A. Guaranteed Capture

The worst case scenario for a guaranteed capture of the drifter is when the drifter is floating tangentially to the communication radius of the robot while the robot is moving away from the drifter. In this case, we can still guarantee the capture of the drifter, only if, the time taken by the robot to complete one circular round is equal to the time taken by the drifter to reach the edge of the communication radius $b$ that is just before escaping the search area. We then define the capture speed of the robot, $s_{c a p}$, as follows:

$$
\begin{equation*}
s_{c a p}=\frac{b s_{r}}{2 \pi r} \tag{5}
\end{equation*}
$$

If the speed of the drifter is $s_{d}$, then we need $s_{d}<s_{c a p}$, for a guaranteed capture. Specifically,

$$
\begin{equation*}
s_{r}>\frac{2 \pi r s_{d}}{b} \tag{6}
\end{equation*}
$$

The guaranteed capture speed of the robot given the drifter speed can be substituted in Eq. 4 to obtain the total time for a guaranteed capture.

## B. Guaranteed Minimum Time Capture

The time to capture the drifter, from Eq. 4, can be minimized, when, $i=\left(n_{s}-1\right)$, such that,

$$
\begin{equation*}
\tau_{\min }=\frac{2 \pi n_{s}}{s_{r}}\left(r-\left(n_{s}-1\right) b\right) \tag{7}
\end{equation*}
$$

From Eq. 1 we know that, $r=n_{s} b$, which we can substitute in the above equation to obtain the following:

$$
\begin{equation*}
\tau_{\min }=\frac{2 \pi b n_{s}}{s_{r}} \tag{8}
\end{equation*}
$$

Hence, to guarantee minimum time to capture, the robot should start with initial radius, $r_{\text {min }}=b$ and incrementally expand the circular radius. This gives us a circular pattern with an increasing radius.

## III. CONTROLLED SIMULATION

## A. Setup

We validated our analytical results on a simulator that we developed for our field trials. We pre-selected a circular search region in the Caribbean sea for all our simulated trials. The initial location of the drifter is within the search region and was selected based on three probability density functions. These are: uniform, triangular and $|x|$ distributions. The uniform distribution provides starting locations that are unbiased in the search region whereas, the triangular distribution gives the locations that are biased in the inner rings of the spiral and the $|x|$ distribution provides biased locations in the outer rings of the spiral. The triangular and $|x|$ distributions were specifically selected to illustrate the two extreme cases where outward spiral and inward spiral, respectively, performed better than the other. We also varied the communication radius to study its effect on the capture time.

## B. Results

We recorded the time to capture the target and number of successful captures, given the initial distribution, the communication radius and the spiral search pattern for each trial of our experiment. The average capture time per 1000 trials is presented in Fig. 2. In the results for uniform distribution it can be observed that outward spiral performed better than the inward spiral for all communication ranges. However, the the outward spiral missed the target more often (1.3\%) than the inward spiral. These results are consistent with our previous work presented in [3]. In case of triangle distribution, since the initial locations are biased towards the spiral center, the outward spiral performed better for search time. This trend was reversed for the $|x|$ distribution where
the inward spiral provided shorter search time and also more captures $(1.9 \%)$ than the outward spiral.


Fig. 2: Initial locations of the drifter are simulated with uniform distribution, triangle and $|x|$ distributions.

## IV. CONCLUSIONS

In this paper, we provided analytical results for spiral search patterns to obtain guaranteed capture and minimum capture time of lost targets. We presented simulation results which validate our hypothesis that inward spiral patterns provide guaranteed capture whereas outward spirals minimize the capture time. The results are consistent for the three probability density functions representing the distribution of initial location of the drifter.

## REFERENCES

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[^0]:    The authors are with the Centre for Intelligent Machines, McGill University, Montréal, Québec, Canada. email:\{malika, dudek\}@cim.mcgill.ca

