Questions

1. Consider family of horizontally oriented V1 simple cells which we model as Gabors of the form

$$\sin(\frac{2\pi}{N}k_1\ y)\ G(x,y,\sigma)$$

where $G()$ is a Gaussian.

   (a) Sketch the receptive field profile of this family of cells, assuming that the Gaussian window is a bit bigger than one cycle of the sinusoid.

   (b) Sketch the linear responses of this family of cells to an image that consists of a bright square on a dark background. Assume the bright square is much larger than the envelope of the cell. In your sketch, it is enough to indicate image regions with positive, negative, and zero responses.

   (c) Same question as (b), but now the cells are complex rather than simple. (Complex cells will be covered in lecture 6.)

2. Suppose we define a Gabor whose spatial frequency is twice that of the previous question, namely $2k_1$. How should the σ change so that the new Gabor has the same shape as the one in the previous question and differs only in the overall size.

3. What is the spatial frequency of the 2D sinusoid: $\sin(\frac{2\pi}{N}(3x+4y))$ in units of cycles per N pixels? Here I am asking for the spatial frequency in the direction in which the sinusoid is varying most, namely perpendicular to the direction in which the sinusoid is constant.

4. What is the spatial frequency of the 2D sinusoid: $\sin(\frac{2\pi}{N}(k_0x+k_1y))$ in units of cycles per N pixels? Again, I am asking for the frequency in the direction in which the values are changing most.

5. A sine function is related to a cosine function by a shift ϕ in *phase angle*.

$$\sin(\frac{2\pi}{N}k_0x) = \cos(\frac{2\pi}{N}k_0x+\phi).$$

How many pixels does this shift ϕ correspond to ?

6. The figure on p. 7 of the lecture notes shows the linear response of a vertically oriented cosine and a sine Gabor cell as a function of the x position x_{line} of an image stimulus line. For the cosine Gabor, this linear response is by definition the inner product of the Gabor template with the image of the line:

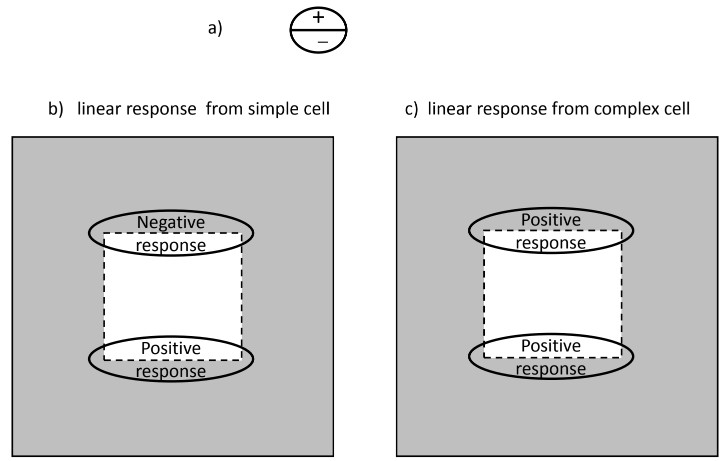
$$< cosGabor(x,y,...), I(x,y;x_{line}) > \ = \sum_{(x',y')} cosGabor(x',y',...) I(x',y';x_{line})$$

A few points about notation: I have simply written ... in the cos Gabor argument, instead of writing the k_x, etc parameters, but you should keep in mind that there are parameters there; The $<\ \ >$ notation here is for inner product; $I(x',y';x_{line})$ has value 0 everywhere except on $x = x_{line}$ where we can assume it has value 1.

Now finally the question: Simplify the summation on the right side above and show that this linear response has a 1D cosine Gabor profile.

Solutions

1. For the response from the complex cell in (c), wait for lecture 6.



Responses are zero outside of the two ellipsoidal regions shown.

2. Twice the spatial frequency means half the wavelength, so the σ for the new Gabor be half as great, namely $\sigma/2$. (Note that a Gaussian function has an integral of 1, so shrinking the σ will double the height of the Gaussian. So one has to be careful when saying that the bigger and smaller Gaussians have the same shape.)

3. The spatial frequency is $\sqrt{3^2+4^2} = 5$ cycles per N pixels. One way to see this is that if you go from (0,0) to $(3N, 4N)$, which is a distance of $5N$, then the argument of the sine goes from 0 to $25 * 2\pi$, so you will have cycled 25 times. So, you'll have gone 25 cycles for a distance of $5N$, or 5 cycles per N pixels.

4. Using the same argument as in the previous question, the answer is $\sqrt{k_0^2 + k_1^2}$.

5. The argument of the sine and cosine functions are in radians, and so $\phi = -\frac{\pi}{2}$ since the cosine needs to be shifted by 90 degrees to the right to get the sine function, i.e. $f(x-a)$ shifts the function to the right when $a > 0$.

$$\sin(\frac{2\pi}{N}k_0 x) = \cos(\frac{2\pi}{N}k_0 x - \frac{\pi}{2}).$$

How many pixels does this shift correspond to? We can rewrite the cosine as

$$\cos(\frac{2\pi}{N}k_0(x - \frac{N}{4k_0}))$$

and we see that the shift is $\frac{N}{4k_0}$ pixels. This makes sense because each period of the sinusoid is N/k_0 pixels – that is, the sinusoid has k_0 cycles over N pixels – and 90 degrees is 1/4 of one period, thus $\frac{N}{4k_0}$ pixels.

6. $I()$ only depends on x position and so the y part can be pulled out of the x sum. Then we can replace $I(x'; x_{line})$ with the value 1 and plug in $x' = x_{line}$ into the cosGabor argument, since $I(\ )$ is only non-zero at that one value.

$$\begin{aligned}
\sum_{(x',y')} cosGabor(x', y', ...)I(x', y'; x_{line}) &= \sum_{x'} I(x'; x_{line}) \sum_{y'} cosGabor(x', y', ...) \\
&= \sum_{y'} cosGabor(x_{line}, y', ...) \\
&= cos(k_x x_{line})G(x_l) \sum_{y'} G(y') \\
&= cos(k_x x_{line})G(x_l)
\end{aligned}$$

since the cosine does not depend on y', and the sum of a Gaussian is 1. Thus we see that the response has the same 1D Gabor profile as the cell itself.