PANTOGRAPH PROJECT

presented

to

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Abstract

The purpose of this project, is to continue on the results achieved by Jehangir Choksi, Gonzalo Layin and Angelo Mirarchi in their Project Laboratory (april 13, 1993): the designed pantograph is the master of a teleoperation system whose slave, which has he position of its master, manoeuvres in a virtual environment. The virtual forces aplied to the slave are reflected to the operator by the way of the manipulandum. Knowing position and velocity should allow one to perform any desired force model. The staility problems occuring when one wants to model string or damping forces will be emphasized.

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Chapter 1

INTRODUCTION

Teleoperation in general enables to improve the control, via a master manipulandum, of a slave teleoperator being in a remote environment, by reflected the contact force to the operator. The pantograph is a special case of teleoperation where the slave (wose control is not a problem, since its position is the master's one, which is detected by sensors) is "evolue" in a virtual environment. The virtual forces applied to the slave are reflected to the operator, thanks to the pantograph.

The first part of this report is concerned with the setting-up of the teleoperation system: the pantograph (the master), its sensors, the Digital Signal Processor (simulating the slave in its virtual world) and the amplifier are connected to each other. I order to improve the precision, the ouput voltages of the sensors are amplified so that they span over the whole range of values allowed by the DSP: the 12 bits of the DSP are indeed used to cover a specified range.

Then the basic routines of the DSP running program are established: the first one changes the sensors output voltages into the position of the manipulandum. Thanks to these latter values and to the desired force model, the forces to be reflected to the oprator are computed and turned into the corresponding torques the motors have to provide. Afterwards, the most difficult part consists in finding the best model required to achieve the desired results, taking into account stability problems.

For instance the implementation of a stiff virtual wall implies stability problems depending on the sampling rate and stiffness of the wall: a delay of one sampling period enhances the difficulty of handling this case. In order to simulate friction, the frst derivative of the position is also required. Because of the sensors' noise, it is necessary to find a filter which would provide a not too noisy velocity, whithout having too much lag.

The implementation of the model must also avoid some unstable cases such as sharp corners which would result in "cycles limites". This is due to the fact that a discrete system may gain some energy where a continuous physical one would not, because of th sampling. Therefore in order to get rid of these phenomena, one has to use smoother corners.

Chapter 2

PRESENTATION OF THE MATERIALS

2.1 DESCRIPTION OF THE SYSTEM

The following sketch (figure 2.1) explains the principle of the discrete controller:
Figure 2.1: CONNECTIONS

The pantograph sensors provide 2 voltages directly proportional to θ_1 and θ_2 the position angles of the arms with respect to the horizontal (see figure 2.2). The dicrete values are then turned in the DSP into the corresponding angles then ositions. Thanks to these data, the forces can be computed according to the force model chosen, and are changed into the corresponding torques. Therefore the voltage ouputs will be the values of the intensities required to achieve the desired torques, sine the amplifier provides a current output proportional to the input voltage. These ouputs are plugged into the pantograph motors.

The dimensions found to be optimal for the pantograph in [1], are summarized in figure 2.2.
Figure 2.2: PANTOGRAPH sketch

The DSP has an accuracy of 12 bits on a range of [-5 +5] or [-10 +10]. Therefore in order to use the maximum precision available, the voltage ouputs of the sensors will have to span over the whole possible range. It can be found from figure 2.2 that each θ angle takes its values in the range [-35.7 87.9]. Therefore only 35
The use of DSP necessarily implies a delay of one sampling period, since the output of time k has been computed using inputs of time k-1.

The amplifier is assumed to be perfect, i.e. it has a transfer function equal to 1. Indeed the electrical time constant is much shorter than the mechanical one. Therefore the amplifier output current is supposed to be equal to the input voltage i.e. a volage of 1V gives a current of 1A.

2.2 THE SENSORS

The sensors are basically potentiometers, and in this case use the power line of the DSP which provides 12V. As the voltage contains some high frequency noises, the signal is first filtered with a low-pass and then regulated at $\pm 8V$. As it has been sen in the previous section, only 35
The resulting electrical circuit is shown in figure 2.3.

Figure 2.3: ELECTRICAL CIRCUIT

Chapter 3

TRANSFORMATON ROUTINES

Some transformation procedures are needed for the DSP to play its role. There are basically 3 of them: the first one enables to get the cartesian positions from the angles whereas the second one turns the forces to reflect to the operator into the correspnding torques. The third one obtains the angles θ_1 and θ_2 from the sensors voltages.

3.1 TRANSFORMATION: $\theta_1,\ \theta_2$ to $x,\ y$

This routine performs the computation of $x,\ y$ in function of $\theta_1,\ \theta_2$ which are available from the sensors.
Refering to figure 2.2, the following equations can be deduced:

$$x_1 = -d/2 - a * cos(\theta_1) \tag{3.1}$$
$$y_1 = a * sin(\theta_1) \tag{3.2}$$
$$x_2 = d/2 + a * cos(\theta_2) \tag{3.3}$$
$$y_2 = a * sin(\theta_2) \tag{3.4}$$

$$(x_c - x_1)^2 + (y_c - y_1)^2 = b^2 \tag{3.5}$$
$$(x_c - x_2)^2 + (y_c - y_2)^2 = b^2 \tag{3.6}$$

Where 3.1 to 3.4 simply represent the projection of points A, B on the axes. 3.5 and 3.6 state that point C is on circles of radius b centered on point A and B. Substracting 3.6 to 3.5, one can get:

$$x_c = -\overbrace{\frac{y_2 - y_1}{x_2 - x_1}}^{A_1} y_c + \overbrace{\frac{y_2^2 - y_1^2}{2(x_2 - x_1)} + \frac{x_1 + x_2}{2}}^{A_2} \tag{3.7}$$

Plugging this in 3.5, the following result comes out:

$$\begin{aligned} y_c &= \frac{-\left(2A_1(A_2-x_1)-2y_1\right)+\sqrt{\left(2A_1(A_2-x_1)-2y_1\right)^2-4(a_1^2+1)\left((A_2-x_1)^2+y_1^2-b^2\right)}}{2(A_1^2+1)} \\ x_c &= A_1 y_c + A_2 \end{aligned} \tag{3.8}$$

The equations 3.8 give in a close-form solution the desired result:

$$\begin{aligned} x_c &= f_1(\theta_1,\theta_2) \\ y_c &= f_2(\theta_1,\theta_2) \end{aligned} \tag{3.9}$$

3.2 TRANSFORMATION: $F_x,\ F_y$ to $\Gamma_1,\ \Gamma_2$

Thanks to the force model and to the known position (velocity may also be necessary), one knows the vector force to reflect to the operator. Therefore the torques (to apply to the motors) which would lead to the desired force must be computed.

The next equations refere to figure 2.4 where it can be noticed that θ_1 is consistent with figure 2.2. In order to keep symetry property θ_1 is negative in the geometric way.

$$\gamma_1 = arctan2(y_c-y_1;x_c-x_1) \tag{3.10}$$

$$\beta_1 = \theta_1+\gamma_1 \tag{3.11}$$

$$\alpha_1 = \frac{\pi}{2}-\beta_1 \tag{3.12}$$

Figure 2.4

With these relations, F_1 can be deduced from Γ_1 thanks to the relation $F_1=\Gamma_1/d=\Gamma_1/(acos(\alpha_1))$. Therefore it comes out that:

$$\vec{F_1} = \frac{\Gamma_1}{acos(\alpha_1)}\begin{bmatrix} cos(\gamma_1) \\ sin(\gamma_1) \end{bmatrix} \tag{3.13}$$

Using symmetry, the following relations are directly deduced from the previous ones. It should be noticed that 3.14 and 3.17 are slightly different:

$$\gamma_2 = arctan2(y_c-y_2;x_2-x_c) \tag{3.14}$$

$$\beta_2 = \theta_2+\gamma_2 \tag{3.15}$$

$$\alpha_2 = \frac{\pi}{2}-\beta_2 \tag{3.16}$$

$$\vec{F_2} = \frac{\Gamma_2}{acos(\alpha_2)}\begin{bmatrix} -cos(\gamma_2) \\ sin(\gamma_2) \end{bmatrix} \tag{3.17}$$

Therefore the reflected force is:

$$\vec{F}=\vec{F_1}+\vec{F_2}=\frac{1}{a}\begin{bmatrix} \frac{cos(\gamma_1)}{cos(\alpha_1)} & -\frac{cos(\gamma_2)}{cos(\alpha_2)} \\ \frac{sin(\gamma_1)}{cos(\alpha_1)} & \frac{sin(\gamma_2)}{cos(\alpha_2)} \end{bmatrix}\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \tag{3.18}$$

And the torques to apply to get this force are:

$$\begin{bmatrix} \Gamma_1 \\ \\ \Gamma_2 \end{bmatrix} = \frac{a}{sin(\gamma_1+\gamma_2)} \begin{bmatrix} sin(\gamma_2)cos(\alpha_1) & -sin(\gamma_1)cos(\alpha_2) \\ \\ cos(\gamma_2)cos(\alpha_1) & cos(\gamma_1)cos(\alpha_2) \end{bmatrix} \begin{bmatrix} F_1 \\ \\ F_2 \end{bmatrix} \tag{3.19}$$

It also interesting to notice that from these relations, it is straightforward to obtain the jacobian matrix. Indeed 3.18 provides M such that $F = M\Gamma$ and the jacobian is defined by $v = J_v\omega$ where v and ω are respectively linear ad angular velocities. Then applying the energy conservation theorem expressed in equation 3.20, the result is given in equation 3.21:

$$\Gamma^T\omega = F^T v \tag{3.20}$$
$$J_v = M^{-T} \tag{3.21}$$

3.3 DETERMINATION OF THE ANGLES

The sensors provide 2 voltages directly proportional to θ_1 and θ_2. The problem is to determine $a_1,\ b_1,\ a_2,\ b_2$ of eq. 3.22:

$$\begin{aligned} \theta_1 &= a_1V_1 + b_1 \\ \theta_2 &= a_2V_2 + b_2 \end{aligned} \tag{3.22}$$

In order to determine these values with the best possible accuracy, 4 pairs of voltages are measured in the corners P1, P2, P3 and P4 (see figure 2.2) of the work space, where the corresponding angles can be computed. Then a Least-Square is applied to tret these data and to find the best result:

The relations between voltages and angles are:

$$\begin{aligned} \theta(1) = aV(1) + b + e(1) \\ \theta(2) = aV(2) + b + e(2) \\ \theta(3) = aV(3) + b + e(3) \\ \theta(4) = aV(4) + b + e(4) \end{aligned} \tag{3.23}$$

Where $\theta(i) = \theta_{P_i}$, $V(i) = V_{P_i}$ and e(i) are the sensors noises assumed to be zero mean.

The LS solution is expressed in eq. 3.24

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{4\sum_{i=1}^{4} V(i)^2 - \left(\sum_{i=1}^{4} V(i)\right)^2} *$$

$$\begin{bmatrix} 4\sum_{i=1}^{4} V(i)\theta(i) - \left(\sum_{i=1}^{4} V(i)\right)\left(\sum_{i=1}^{4} \theta(i)\right) \\ -\left(\sum_{i=1}^{4} V(i)\theta(i)\right)\sum_{i=1}^{4} V(i) + \left(\sum_{i=1}^{4} V(i)^2\right)\sum_{i=1}^{4} \theta(i) \end{bmatrix} \tag{3.24}$$

Chapter 4

ACQUISTION OF POSITION AND VELOCITY

Chapter 5

IMPLEMENTATION OF A VIRTUAL WALL

5.1 MODEL OF THE SYSTEM

The easiest way to simulate a wall is to produce a force proportional to the displacement with repect to the position of the wall, as shown in figure 5.1.

Figure 5.1

The gain K must be large enough for the virtual wall to feel like a wall. In [2] a stiffness of 2000-8000 N/m is recommended. A delay of one period is introduced in the loop since the computation time of the force F in the processor must be taken nto account.
Then one also needs a model for the operator which can be represented as follows (figure 5.2):

Figure 5.2

This leads to the transfer function:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + k} \tag{5.1}$$

where coefficients f and k can vary in function of the way the operator is holding the manipulandum. k may vary from 0 to some k_{max}, while f has a minimum value equal to the friction of the manipulor itself.

5.2 STABILITY CONDITIONS

As the sampling period T_s is very small (1E-3), it can be assumed that the behaviour of the system in the discrete case is roughly the same as in the continuous case. Therefore the stability conditions found in this latter case should also be correct i the former case. The ZOH is approximated by a delay of half a sampling period ($T_s/2$), which means that T (see Figure 5.3) is equal to the time delay plus $T_s/2$.

5.2.1 WITHOUT FRICTION CASE

In this case, the close-loop system becomes:

Figure 5.3

e^{-Ts} can be expanded at the first order, using Pade approximation. The open-loop then becomes:

$$TF_{OL} = K\frac{1 - s\frac{T}{2}}{\left(1 + s\frac{T}{2}\right)(Ms^2 + fs + k)} \tag{5.2}$$

The characteristic equation is therefore:

$$\frac{MT}{2}s^3 + \left(M + f\frac{T}{2}\right)s^2 + \left(f + k\frac{T}{2} - K\frac{T}{2}\right)s + k + K = 0 \tag{5.3}$$

Using Routh criterion to get stability conditions gives the following Routh array:

$$\begin{array}{ll}
\frac{MT}{2} & \frac{-KT}{2} + f + \frac{kT}{2} \\[2ex]
\overbrace{M + \frac{fT}{2}}^{A} & k + K \\[2ex]
\overbrace{\frac{1}{A}\left(A\left(\frac{-KT}{2} + f + \frac{kT}{2}\right) - (K+k)\frac{TM}{2}\right)}^{B} & \\[2ex]
\text{K+k} &
\end{array}$$

So the stability condition i.e. $B > 0$ results in:

$$K < \frac{Mf + f^2\frac{T}{2} + kf\frac{T^2}{4}}{MT + f\frac{T^2}{4}} \tag{5.4}$$

As $T << 1$ and assuming that $M,\ k,\ f$ are of the same order, the stability condition turns out to be :

$$K < \frac{f}{T} \tag{5.5}$$

With a time delay of one sampling period T_s, the variable T is equal to $2T_s/3$, and the stability condition becomes:

$$K \ < \ \frac{2}{3}\frac{f}{T_s} \tag{5.6}$$

5.2.2 WITH FRICTION CASE

In this paragraph, the wall is not any more represented by $F = -Kx$ but $F = -Kx - B\dot{x}$. In this case the Open-Loop transfer function turns out to be:

$$TF_{OL} = \frac{(K + Bs)\left(1 - s\frac{T}{2}\right)}{\left(1 + s\frac{T}{2}\right)(Ms^2 + fs + k)} \tag{5.7}$$

which gives the characteristic equation:

$$\frac{MT}{2}s^3 + \left(M + f\frac{T}{2} - B\frac{T}{2}\right)s^2 + \left(f + k\frac{T}{2} - K\frac{T}{2} + B\right)s + k + K = 0 \tag{5.8}$$

So the routh array is:

$$\begin{array}{ll}
\frac{MT}{2} & \frac{-KT}{2} + f + B + \frac{kT}{2} \\
\overbrace{M + \frac{fT}{2} - \frac{BT}{2}}^{A} & k + K \\
\overbrace{\frac{1}{A}\left(A\left(\frac{-KT}{2} + f + B + \frac{kT}{2}\right) - (K + k)\frac{TM}{2}\right)}^{C} & \\
\text{K+k} &
\end{array}$$

The first condition to satisfy, $A > 0$, implies equation 5.9. And the stability condition, $C > 0$ is equivalent to equation 5.10:

$$B < f + \frac{2M}{T} \tag{5.9}$$

$$K < \frac{M(f + B) + (f^2 - B^2)\frac{T}{2} + k(f - B)\frac{T^2}{4}}{MT + (f - B)\frac{T^2}{4}} \tag{5.10}$$

With the assumptions made in the previous paragraph, these equations can be changed into:

$$B < f + \frac{2M}{T} \tag{5.11}$$

$$K < \frac{f + B}{T} \tag{5.12}$$

And with a time delay of one sampling period, the conditions are the following:

$$B < f + \frac{4}{3}\frac{M}{T_s} \tag{5.13}$$

$$K < \frac{2}{3}\frac{f + B}{T_s} \tag{5.14}$$

5.3 SIMULATION

In order to verify the stability relations of eq. 5.6 and 5.14 the discrete system has been simulated with SIMULINK. The time delay has been fixed to one sampling period T_s.

5.3.1 WITHOUT DAMPING CASE

Figure 5.3.1 represents the diagram realized on simulink for the discrete case. F_0 is the force kx_0 defined in the relation describing the system i.e. $M\ddot{x} + f\dot{x} + k(x - x_0) = F$.

Figure 5.4: continuous diagram

The purpose was to determine the stability zone for K with fixed values of k and f. The sampling period T_s is equal to 1.10^{-3} s. The limit of stability is supposed reached when the oscillations are neither increasing nor decreasing.

For instance Figure 5.5 was obtained with $M = 1$, $k = 100$, $F_0 = 100$, $f = 3$. 3 values for K have been tried:

- For $K = 1500$ (and in fact for any $K < 2000$), one can noticed that the oscillations are decreasing. Therefore the system is stable.
- For $K = 2500$ (and for any $K > 2000$), the system becomes unstable since the amplitude of the oscillations is increasing.
- For the specific value $K = 2000$, the limit of stability has been reached: indeed the oscillations remain the same and the system is neither converging nor diverging.

Figure 5.5

Therefore with these parameter values, the stability zone for K is: $K < 2000$. It is also interesting to notice that this is the value predicted by equation 5.6 i.e.

$$K < \frac{2}{3}\frac{f}{T_s} = \frac{2}{3}\frac{3}{.001} = 2000$$

The results found with different values of parameters f and k are shown below:

RESULTS FOR $M = 1$, $f = 3$ and $T_s = .001$ s			
value of k	experimental stability limit found* for K	theorical stability limit for K with equation 5.6	theorical stability limit for K with equation 5.4
10	1950	2000	2002
100	2000	2000	2002
1000	2000	2000	2003
10000	2000	2000	2013

table 5.3.1: *with a precision of 50

RESULTS FOR $M = 1$, $f = 30$ and $T_s = .001$ s			
value of k	experimental stability limit found* for K	theorical stability limit for K with equation 5.6	theorical stability limit for K with equation 5.4
10	20150	20000	20222
100	20150	20000	20222
1000	20150	20000	20223
10000	20250	20000	20333

table 5.3.2: *with a precision of 50

These tables show that these experimental results are really similar to those found in theory. Therefore one may consider that equation 5.5 provides a rather good value of the upper bound for parameter K to keep stability (with a relative error nferior to 2% in the simulations made). It is also interesting to notice that parameter k does not influence very much the stability limit.

5.3.2 WITH DAMPING CASE

Figure 5.6 is the SIMULINK diagram made in order to simulate a force of the kind $F = -Kx - B\dot{x}$.

Figure 5.6

Tables 5.3.3 and 5.3.4 show how the stability limit evolves in function of parameter k. One can see that it is not of great influence. It can also be noticed that experimental and theorical results are really similar. Indeed the relative error for both ables is around 2.5%

RESULTS FOR $M = 1$, $f = 2$, $B = 4$ and $T_s = .001$ s		
value of k	experimental stability limit found* for K	theorical stability limit for K with equation 5.14
0	3950	4000
10	3950	4000
100	3950	4000
1000	3900	4000
10000	3900	4000

table 5.3.3: *with a precision of 50

RESULTS FOR $M = 1$, $f = 40$, $B = 20$ and $T_s = .001$ s		
value of k	experimental stability limit found* for K	theorical stability limit for K with equation 5.14
0	39000	40000
10	39000	40000
100	39000	40000
1000	39000	40000
10000	39000	40000

table 5.3.4: *with a precision of 500

The following simulation was made in order to check the validity of equation 5.14 with variations of parameter f. The worse error in this array is 1.5% which is still a good result.

RESULTS FOR $M = 1$, $k = 100$, $B = 10$ and $T_s = .001$ s			
value of f	experimental stability limit found* for K	theorical stability limit for K with equation 5.14	relative error
10	13333	13300	.5%
20	20000	19700	1.5%
30	26666	26500	.8%
40	33333	33000	1%
50	40000	39400	1.5%

table 5.3.5: *with a precision of 100

5.4 CONCLUSION

The first remark to say is that equations 5.5 and 5.12 are quite good approximations of the stability limit, with an accuracy of 2.5%.

It is also interesting to notice that introducing some damping in the system allows much larger values for the gain of the wall. And for the same value of gain K, a system with friction would oscillate less than without friction.

For instance simulation of figure 5.7 shows very well that the oscillations are all the more small as the friction coefficient gets large.

Figure 5.7: $M = 1$, $f = 30$, $k = 100$, $K = 18000$

Bibliography

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