Limit Cycle Characterization, Existence and Quenching In the Control of a High Performance Hydraulic Actuator

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Abstract

The characteristics and the nonlinear dynamics of a high performance hydraulic actuator produced by ASI Inc. are described and modeled. When a feedback is applied for the regulation of output force, a limit cycle is observed. The existence of the limit cycle can a priori be attributed to one, or to a combination of, the four dominant nonlinear effect that were identified in these actuators. In order to pinpoint its origin, successive approximations are made to apply the describing function principle, so as to predict the onset of the limit cycle as function of the feedback gain. Given the experimental data, this method allows us to attribute beyond any doubt its origin to the electromagnetic hysteresis in the valve, which is based on jet-pipe technology. A multiple term lead-lag controller is designed and implemented to quench the limit cycle and improve the rise time of the force control by more than an order of magnitude.

1 Introduction

Among the closed loop behaviors of nonlinear systems that cannot be explained by linear theory, perhaps the most important is the self-excited stable oscillation called the limit cycle. The influence of a limit cycle on the steady state performance of a closed loop nonlinear system is an important matter for engineers. Aside from exploiting their existence in the design of oscillators, oscillator servos, or in the purposeful creation of dithering signals, for the most part one aims at quenching an existing limit cycle by altering the damping characteristics of the system, insuring that enough energy is dissipated during a cycle to preclude the creation of new one.

Quenching is necessary because it is well known that an existing limit cycle may persist in the presence of an external input signal and thereby interfere with the desired performance of the system. In the ASI hydraulic actuator, a limit cycle was observed. We first recall the dynamics of the system, and the limit cycle is characterized in a series of experiments. In order to determine its cause, the describing function method is used. Some approximations are necessary to simplify the system. Each nonlinearity is studied separately and their describing functions computed. A lead-lag controller is designed to eliminate the limit cycle and improve force control dramatically.

2 ASI Hydraulic Actuator Model

2.1 Physical Description

The ASI hydraulic actuator (Fig. 1) was designed for robotic applications and has the properties required for accurate control. To design a robust force controller, it is useful to have a good model of the system. It was possible to formulate an analytical model. The following is a summary of the results described in [3,4].

It is a force controlled device driven by a high bandwidth jet-pipe suspension valve. In this type of valve, the oil is forced through a flexure member whose opening faces two orifices connected to each side the actuator's piston. A deviation x_v of this member causes a pressure imbalance on each side of the piston resulting in an output force. The oil which does not circulate in the cylinder returns to the pump. This type of valve has several advantages. There is no friction because of the absence of contact between the operating member and the orifices. The inertia of the moving part is reduced to a minimum. The valve is piggy-backed on the actuator, so no intervening hydraulic line dynamics complicates the dynamic response. The bandwidth is practically determined by the valve and force control is easily carried out.

The force is measured by a flexure element attached to the cylinder with a Hall-effect transducer. This force sensor is a non-zero sensor thus calibration is required. The actuator is controlled by a host computer via a card containing analog circuitry for sensor signals conditioning and simple linear controllers. The calibration and the state variables can also be

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assessed digitally through the card [1]. The actuator itself—valve, cylinder and force sensor—forms a single package. Its mass of about .5 Kg and the maximum thrust is about 1,000 N for a nominal supply pressure of 500 psi.

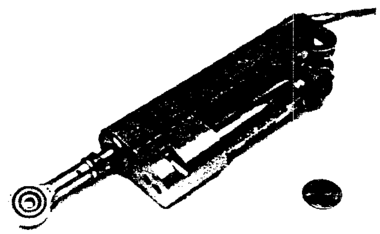

Figure 1: ASI 7/8" bore Integrated Actuator

2.2 Model

Figure 2 shows the block diagram of the closed loop model. The system has four nonlinear elements: hydraulic damping, valve static characteristic, valve hysteresis and friction. Three major linear elements are added to complete an accurate description of the system: valve flexure member, lumped model of fluid dynamics, and sensor transfer function.

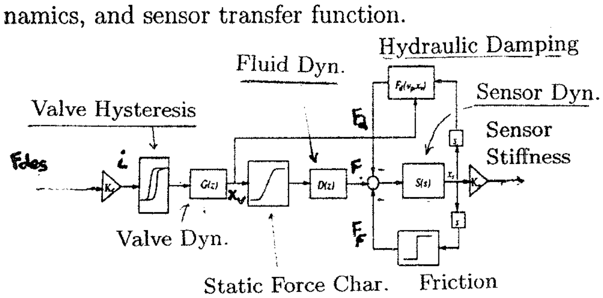

Figure 2: Block Diagram of the model

Hydraulic damping is mostly due to the circulation of oil through the valve orifices. Figure 3 shows the experimental hydraulic damping effect. It increases much faster than linearly with the magnitude of the velocity. When the actuator is backdriven at a velocity larger than its saturation velocity, fluid circulation forces the valve to open causing the damping curves to tapper off. A family of hyperbolic tangents whose magnitudes, scaling and positions with respect to the origin depend on the valve position x_v was fitted to the experimental data.

Hydraulic damping has a **determining** effect on performance since when the velocity is small, the valve opening has a direct impact on force output resulting in high bandwidth force control. When the velocity is

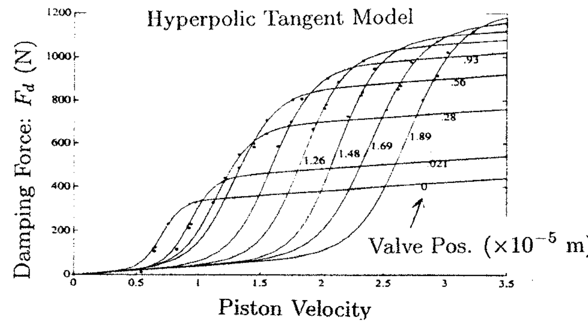

Figure 3: Hydraulic Damping

in the part of the curve where the damping increases exponentially, the stability is increased.

The **valve static characteristic** is the relationship between force and valve opening. For jet-pipe valves, it depends on the geometry and relative position of the orifices. For this design, it is linear until maximum opening is reached, then the actuator saturates. The valve static characteristic is shown Figure 4.

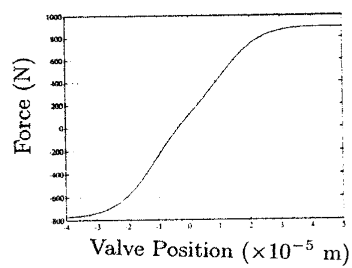

Figure 4: Valve Static Force Characteristic

Hysteresis results from the electromagnetic circuit which drives the valve operation. A plot of hysteresis is given Figure 5.

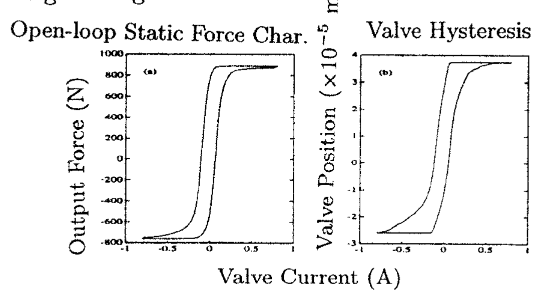

Figure 5: Valve Hysteresis

Solid friction is derived from the contact between the seals and the walls. It was simply modeled by Coulomb friction and stiction. It was found that friction is close to 7 N and stiction is 27 N when pushing on the piston and 15 N when pulling.

The linear characteristics of the valve and the sensor are due to the flexure pipe and to the force sensor attached to the cylinder. The model includes also the lumped fluid dynamics. Some other linear elements could result from some other parts of the system but only those three major ones were retained. The identified dynamics of the valve, the fluid, and the force sensor are given by a 10th order transfer function with two delays. The resulting model is quite precise and was experimentally verified [2].

The open loop bandwidth was found to be 20 Hz. Of course, open loop control is impractical because of the presence of hysteresis and friction. The closed loop bandwidth is about 100 Hz and decreases for higher input amplitudes due to the nonlinear elements, hydraulic damping and valve characteristic in particular.

3 Limit Cycle Existence

3.1 Preliminary Experiments

A limit cycle is defined as an isolated closed curve in the phase plane. The trajectory has to be both closed, indicating the periodic nature of the motion, and isolated, indicating the limiting nature of the cycle with nearby trajectories converging or diverging from it. We first verified the existence of a limit cycle and not a simple stability limit due to resonance. Figure 6 shows a typical step response with a high proportional gain. The shape of the curve indicates clearly that the phenomenon is a limit cycle, with sustained oscillations at around 90 Hz. The oscillations are stable, even if the piston motions are severely perturbed or damped externally, and the oscillations are almost perfectly sinusoidal. The question is now to determine under what conditions a stable limit cycle occurs.

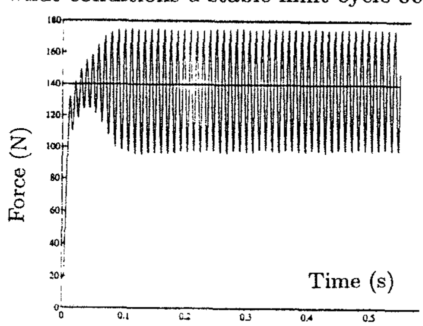

Figure 6: Step response with high gain

A limit cycle must be input dependent: the limit gain and frequency depend on the magnitude of the step input which starts the cycle. Figure 7 shows (with the piston locked at mid-stroke) that when the input increases the gain which gives rise to the onset of oscillation becomes higher while the frequency decreases from 95 Hz to 75 Hz for a range of 0-600 N. The input dependency of the limit cycle is thus established

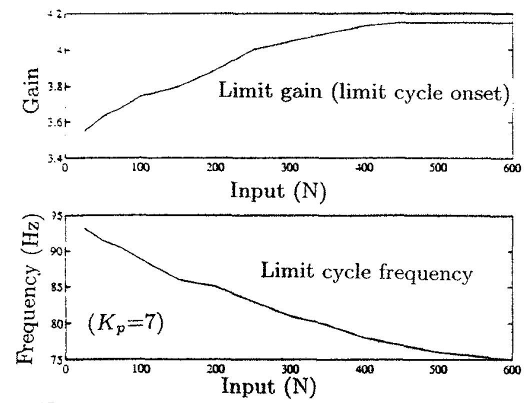

Figure 7: Limit cycle as a function of input

and related to the fact that the closed loop frequency response depends on the input.

These preliminary experiments revealed two major points. Firstly, the limit cycle does not appear randomly but instead can be precisely reproduced under well specified conditions. Secondly, the nonlinearities of the system can all be involved at the onset of the limit cycle, however in the steady state, the effect of each non linearity must be studied. To carry out that study, we used the describing function method.

4 Describing Function Method

Frequency response method is a powerful tool for the analysis and design of linear control systems. It is based on describing a linear system by a complex-value function of the frequency. Some of the strengths of this method lie in the graphical representations used to understand the behavior of a system and to design controllers, but it cannot be directly applied to nonlinear systems. Yet, for some nonlinear systems, an extension of the frequency response method, called the describing function method, can produce good approximations for analysis and prediction of nonlinear behaviors. It can be used reliably for a number of applications such as predicting sub-harmonics, jump phenomena and limit cycles. A describing function is defined as the ratio of the fundamental component of the output of a nonlinear device to the amplitude of a sinusoidal input signal. In general, the describing function depends on the amplitude and frequency of the input signal and is complex because phase shift may occur between the input and the fundamental component of the output. We assume that the input to a nonlinear element is given by: $m(\omega t) = A \sin \omega t$. In general, the steady state output of a nonlinear device can be represented as a Fourrier series: $n(\omega t) = N_1 \sin(\omega t + \phi_1) + N_2 \sin(2\omega t + \phi_2) + \cdots$. We only consider the fundamental component, which in the present case is **experimentally justified** since the shape of the signal is nearly sinusoidal. The describing function becomes: $N(A,\omega) = N_1/Ae^{j\phi_1}$.

The describing function has several interesting properties. When the nonlinearity is single-valued (such as saturation, dead-band ...) there is no phase shift in the output fundamental, $N(A,\omega)$ is real and independent of the frequency ω. In the case of a nonlinearity with memory, corresponding to a double-valued function (such as hysteresis or backlash), there will be a phase associated with the describing function. The Nyquist criterion can be extended to the describing function method.

Consider a closed loop system with unity feedback cascading a nonlinear block $N(A,\omega)$ with a linear one $G(j\omega)$, its characteristic equation is: $1 + G(j\omega)N(A,\omega) = 0$. Self-sustained oscillations of amplitude A and frequency ω will appear if: $G(j\omega) = -1/N(A,\omega)$. This leads to two nonlinear equations (the real part and the imaginary part) in the two variables A and ω. There are usually a finite number of solutions, but since it is generally difficult to solve these equations analytically, graphical approaches are used. Both sides of $G(j\omega) = -1/N(A,\omega)$ are plotted in the complex plane to determine the intersection of the two curves. If the two curves intersect, then a limit cycle should appear and the corresponding values of A and ω are the solution. In order to apply the describing function analysis to our system we must simplify it, although methods exist to deal with problems with several nonlinearities (see [3], [4], [6]), these methods are either difficult to apply to a real system, or quite restrictive. For example, J. Gouws and J.J. Kruger describe in [4] a very interesting method to analyze stability conditions of system with two nonlinearities, but the proposed procedure implies that we can modify the block diagram of the system to put it into a special form, which is not possible here. It was then decided to make approximations in order to study one nonlinear element at a time.

4.1 Simplification of the System

The force static characteristic is single valued and would require, for example, an integrator to generate instability. If we remain in the linear part, we can replace it by a simple gain, so it is unlikely to cause a limit cycle. The damping was first thought to be negligible. If the velocity remains small enough the damping is low. The velocity during a limit cycle was measured for several positions of the piston, inputs and proportional gains. It was quickly realized that limit cycles could be created under conditions that would cause the hydraulic damping to vary drastically during one cycle, often spanning the entire range of the family of curves on Figure 3. Damping is not negligible and can vary according to the valve position from 0 to 1,200 N. The hysteresis and friction were also kept and all those three nonlinearities were studied separately. To simplify the system, other approximations were made. Firstly, the bandwidth of the fluid dynamics, D(z) is around 1000 Hz and its phase is small until 400 Hz. It was replaced by a pure gain since the frequency range of interest is 10 Hz to 200 Hz. Approximate models of the nonlinear elements were derived to apply DFM to friction, damping and hysteresis.

Friction and damping have similar effect on the system and their effects were first investigated. Standard approximate models of the nonlinear elements were used. The describing functions of standard nonlinearities are well known [7], and are not recalled here. A simplified system was built with the hysteresis and the valve characteristic replaced by single gains. The transfer function is then:

$$\frac{F_s}{F_{des}} = \frac{S(s)H(s)}{1 + S(s)H(s) + S(s)sD(A)}$$

where $S(s)$ is the force sensor transfer function, $H(s)$ is the transfer function lumping all other linear elements and $D(A)$ the describing function of either damping or friction. Applying the extension of the Nyquist theorem to this system it can be stated that a limit cycle will appear if:

$$\frac{-1}{D(A)} = \frac{S(s)}{1 + S(s)H(s)}.$$

The Nyquist plot of the right hand side of the equation was plotted Figure 8. The plot of the inverse of the describing function of both damping and friction evidently stick to the real axis and cannot intersect the Nyquist plot of the transfer function except for some extreme gains. The effect of friction and damping can therefore only be marginal. Damping was not

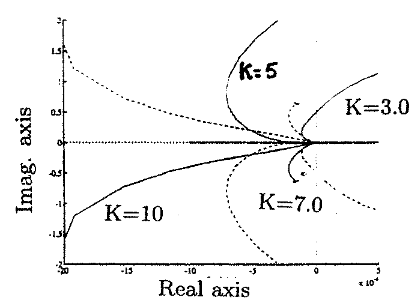

Figure 8: Nyquist plot of RHS (friction and damping)

really expected to generate a limit cycle but the case of friction is different. As shown by W.T. Townsend in [8], the effect of friction on limit cycles is rather complex. The stiction (which was not taken into account here because it only appears at zero velocity and does not change significantly the shape of the describing function) can cause force to enter a limit cycle. Coulomb friction, on the other hand, can extend the system stability bounds but may lead to an input dependent stability. However, in Townsend's experiments, a PI controller was used which added a memory effect through the integrator. In our case a simple proportional controller can cause the system to enter limit cycles. Moreover, during a limit cycle, the piston velocity varies greatly, and the piston is well lubricated and so the friction is reduced. These results show that Coulomb friction and damping cannot maintain a limit cycle even if they can play a role in kicking it off. Indeed, when an experiment is initiated, the piston is not well lubricated and friction and stiction dominate and can give an impulse containing energy at all frequencies which starts the limit cycle.

Hysteresis: Similar to friction and damping, a simplified model of the hysteresis was used and the describing function computed. Several simplified systems were used to study the effect of hysteresis, for example, a constant force was subtracted to account for the effect of damping and friction but the best results (as far as matching the experimental data) were given by the transfer function:

$$\frac{F_s}{F_{des}} = \frac{H(s)S(s)Y(A)}{1 + H(s)S(s)Y(A)}$$

where $Y(A)$ is the describing function of the hysteresis. By applying the extended Nyquist theorem, a limit cycle will appear if:

$$\frac{-1}{Y(A)} = H(s)S(s).$$

The Nyquist diagrams of $H(s)S(s)$ and $-1/N(A)$ were plotted together for various values of the proportional gain. Figure 9 shows an example. When the gain is high enough an intersection certainly occurs which can predict the existence of a limit cycle. The two curves first intersect one another for Kp = 8 at a frequency of 130 Hz. When we increase the gain, the frequency of the intersection point decreases and the amplitude of the limit cycle increases.

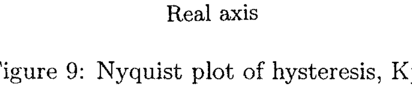

Figure 9: Nyquist plot of hysteresis, Kp=15.

4.2 Discussion

The predicted frequency is higher than found in the experiments. Considering the numerous approximations made in this study, this is hardly surprising. The discrepancy is nonetheless never greater than a factor of two, and the result is qualitatively correct: when the gain increases, the frequency decreases. However, it was found that the amplitude of the limit cycle did not increase as dramatically as predicted by the theory. This is easily explained by the fact that we neglected hydraulic damping which has the property of increasing rapidly with the magnitude of the velocity and therefore has an amplitude self-regulating effect.

It was concluded that the existence of stable and repeatable limit cycle could reliably be attributed to the electromagnetic hysteresis in the valve, excluding all other observed nonlinear elements, although they probably all play a transient role at the onset of the cycles. Moreover, the limit cycle is **not** due to an instability of the feedback loop leading sustained oscillations with amplitudes nearing saturation.

Since limit cycles are considered to be a nuisance for the purpose of these actuators, in the next section, we show that a simple linear controller can be designed to quench them unconditionally and to improve the force control rise time by more than one order of magnitude!

5 Force Control

A step response with proportional control is shown Figure 10 with Kp = 3. The rise time is around 0.045 and the overshoot is about 20 percent. In an attempt to combat the effect of hysteresis with a simple linear control, it was proposed that this effect was equivalent to introducing a phase lag at the critical crossover frequency. No rigorous design method was used, but from the previous observation, a trial and error method was attempted. To compensate for lag, a lead compensator of the form $K(\alpha Ts + 1)/(Ts + 1)$ ($\alpha = 130$, $\phi_{max} = 80^o$ for maximum action in the region 60 to 100 Hz) was put on the error path for a more damped response. A lag term with a 0.5 s time constant was cascaded to get the desired accuracy at low frequency. Such lag terms were found to produce excellent results for friction compensation in lubricated mechanical systems. The resulting compensator produced a step response shown Figure 11. The rise time from 10% to 90% is 0.01 s and the overshoot is about 14%, while the steady state error is less than 2N (1 part in 500).

Then we proceeded to adding more lead terms. A second order lead controller was experimented with and then a fourth order one. After some tuning to avoid actuator saturation for high desired inputs, the compensator produced the result shown Figure 12. Limit cycles could not be made to appear under any condition and the performance results are summarized in the table 1.

These interesting results can be explained by the robustness properties of elementary lead-lag compensators. In effect, more than 180° of phase lead action is applied over two decades by a four term lead. The obvious tradeoff is the high energy demand in the error signal versus the admissible dynamic range of the actuator. A more rigorous proof of the validity of the method may involve the use of the small gain theorem [9,10]. It is hoped that the methods described in this paper can be adapted to other robotic actuator systems having comparable characteristics.

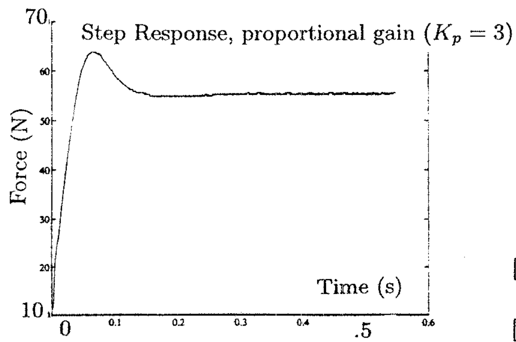

Figure 10: Step response with proportional control

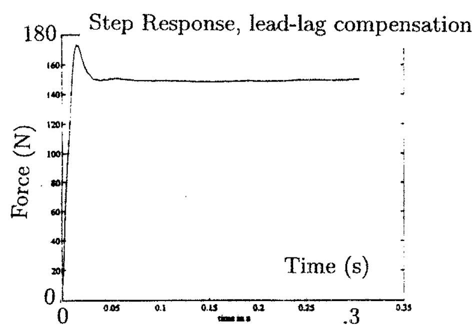

Figure 11: Step response with lead-lag control

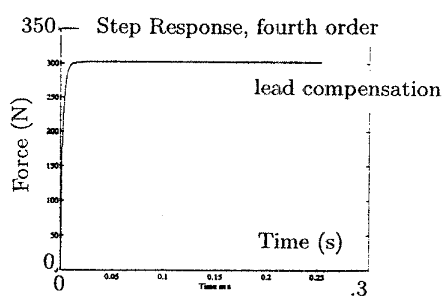

Figure 12: Step response with 4th order lead

	Prop.	L-L	4th order
Rise Time 10 to 90%	0.04 s	0.01 s	0.007 s
Overshoot	20 %	14 %	0 %
Settling Time	0.18 s	0.08 s	0.04 s
Steady State Error	6 N	< 2 N	< 2 N

Table 1: Summary of Force Control Results

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