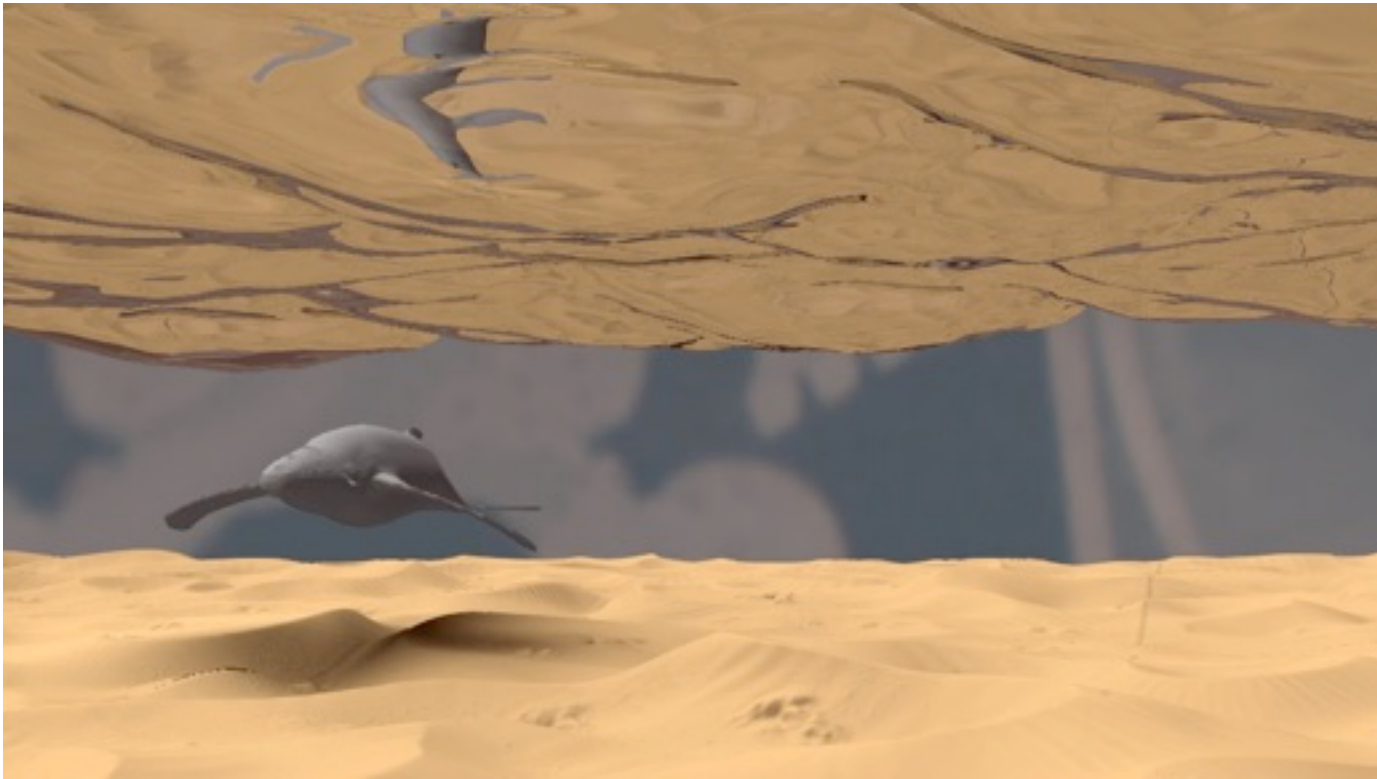


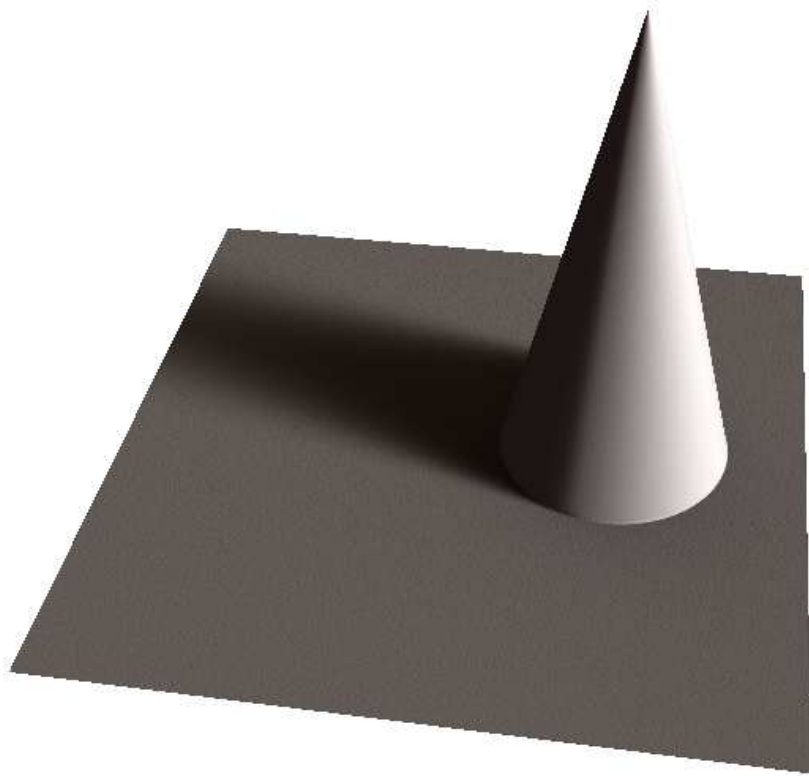
Practical Shading of Height Fields and Meshes using Spherical Harmonics Exponentiation



Aude Giraud

Derek Nowrouzezahrai

Goals & Motivation



[SN08]

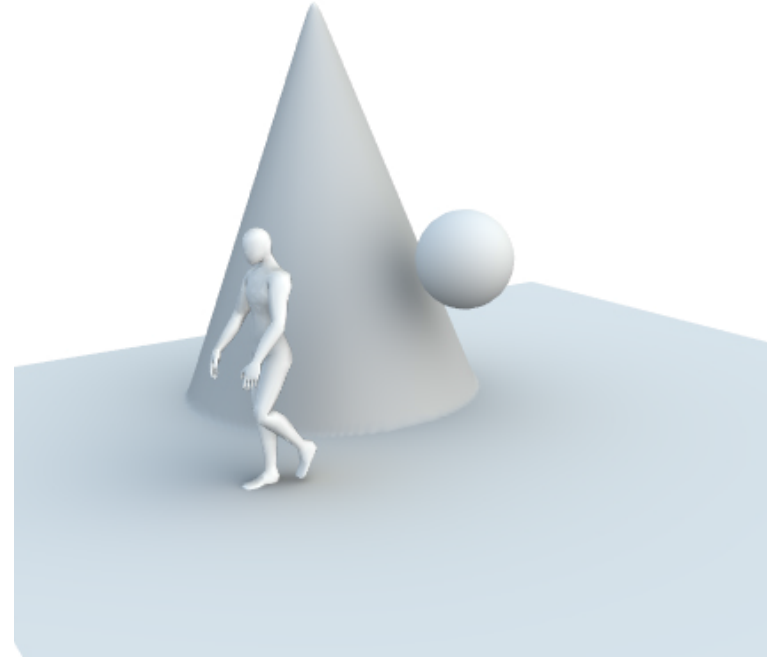


[RWS*06;SGNS07]

Goals & Motivation



[RWS*06;SGNS07]



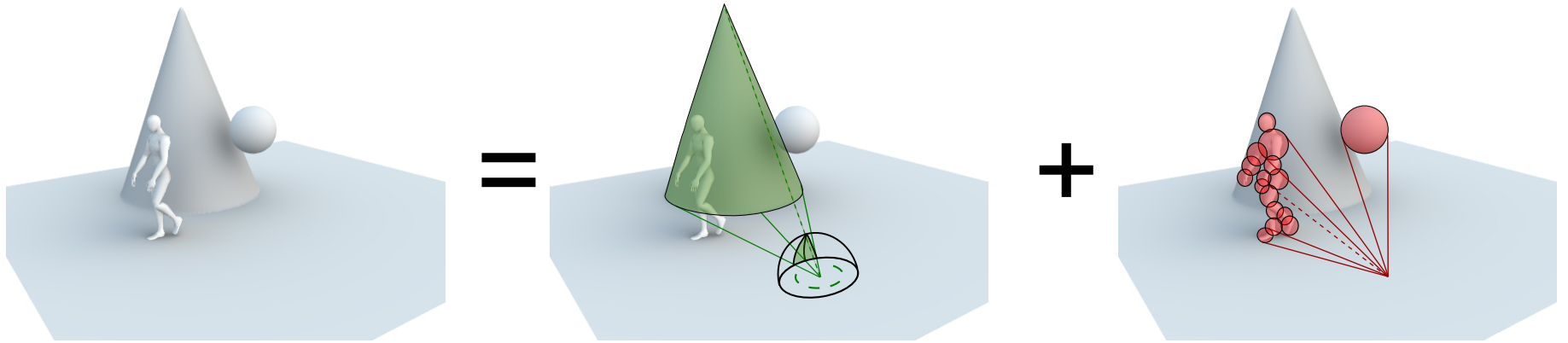
Our results

Contributions

- unifying SH exponentiation on HFs and meshes
 - dynamic geometry and HF visibility (no precomputation)
 - diffuse and glossy BRDFs in log SH

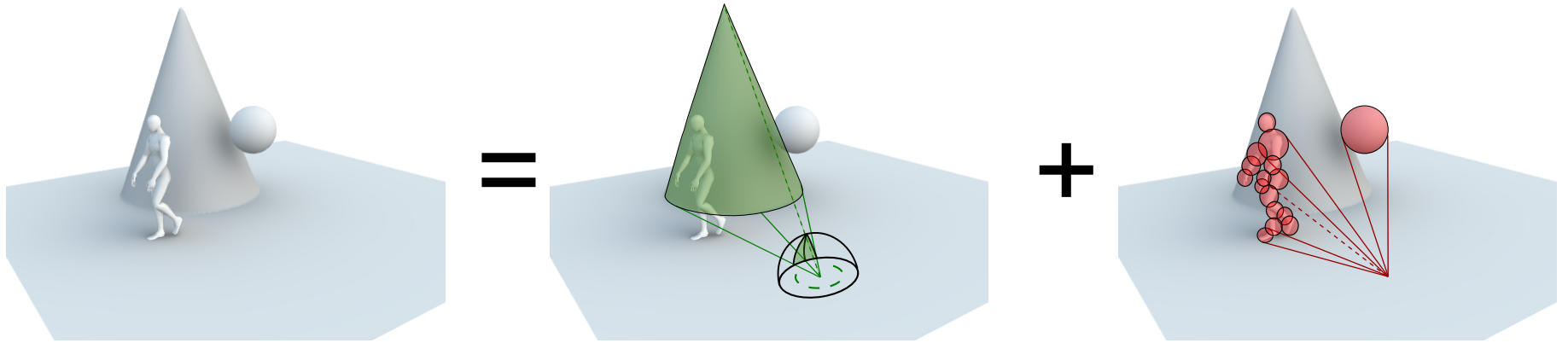
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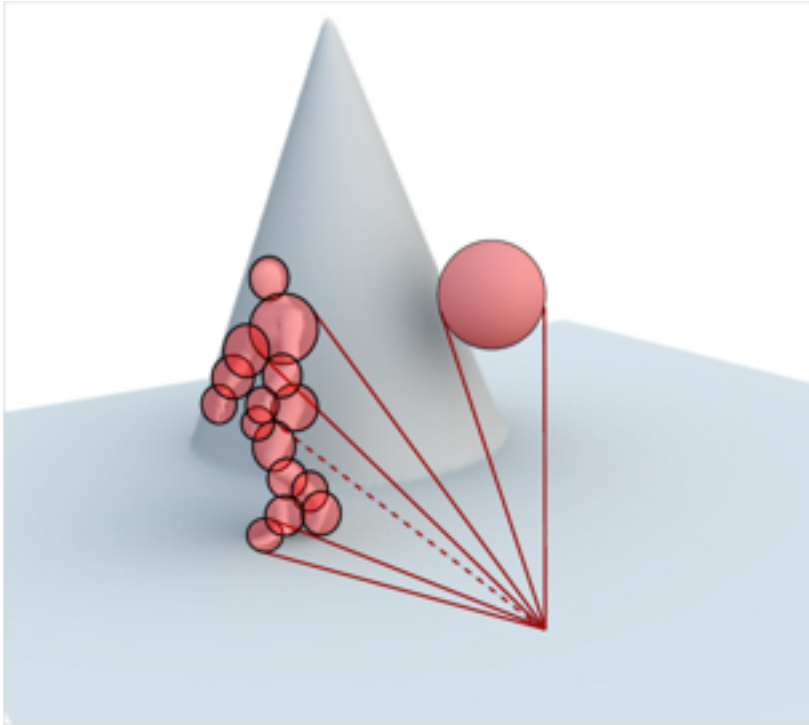
- real-time performance and simple implementation
- limitation: only ***soft direct illumination***
- applications:
 - landscape rendering (flight simulators, mapping/navigation)
 - interactive gaming

Accumulating Log-SH Visibility

Given spherical log-SH visibility for

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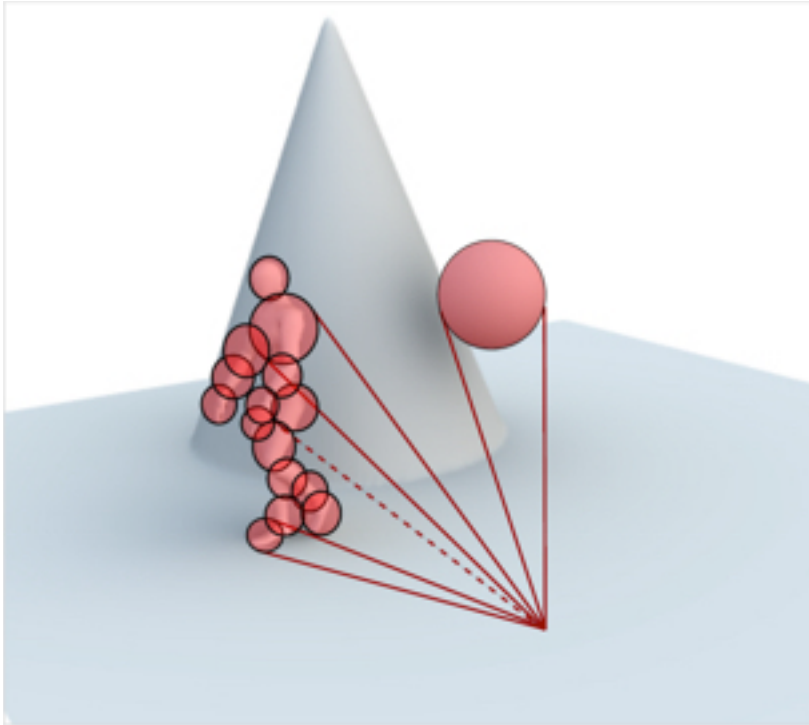


dynamic blocker “meshes”

$$\{ \mathbf{v}_{\log}^0, \mathbf{v}_{\log}^1, \dots, \mathbf{v}_{\log}^{B-1} \}$$

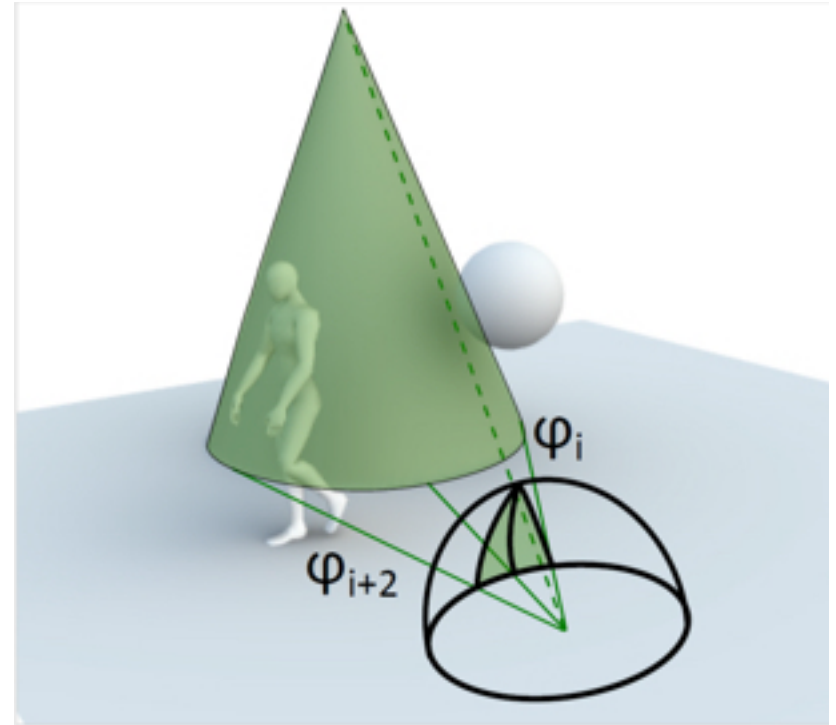
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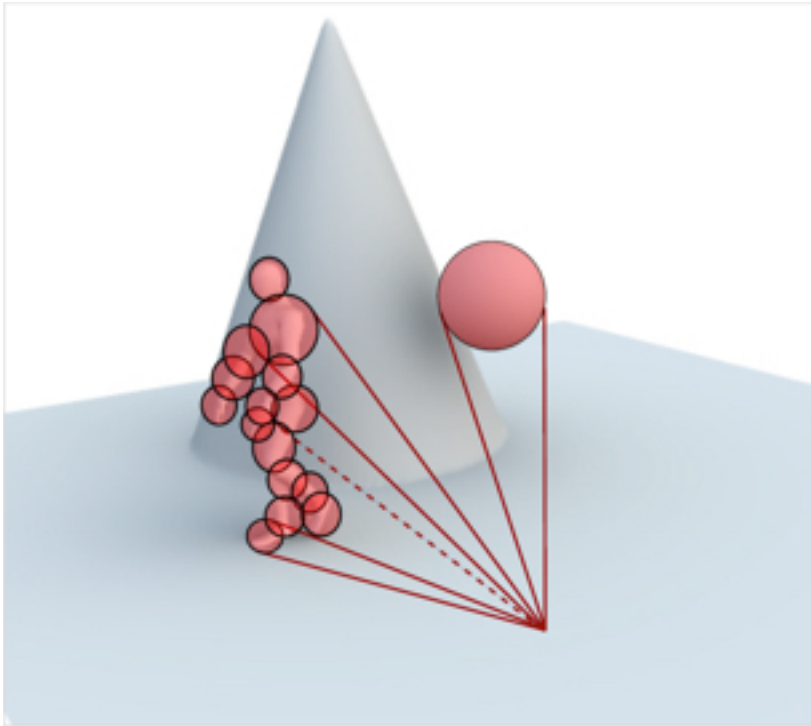


dynamic height field geometry

$$\mathbf{v}_{\log}^{\text{HF}}$$

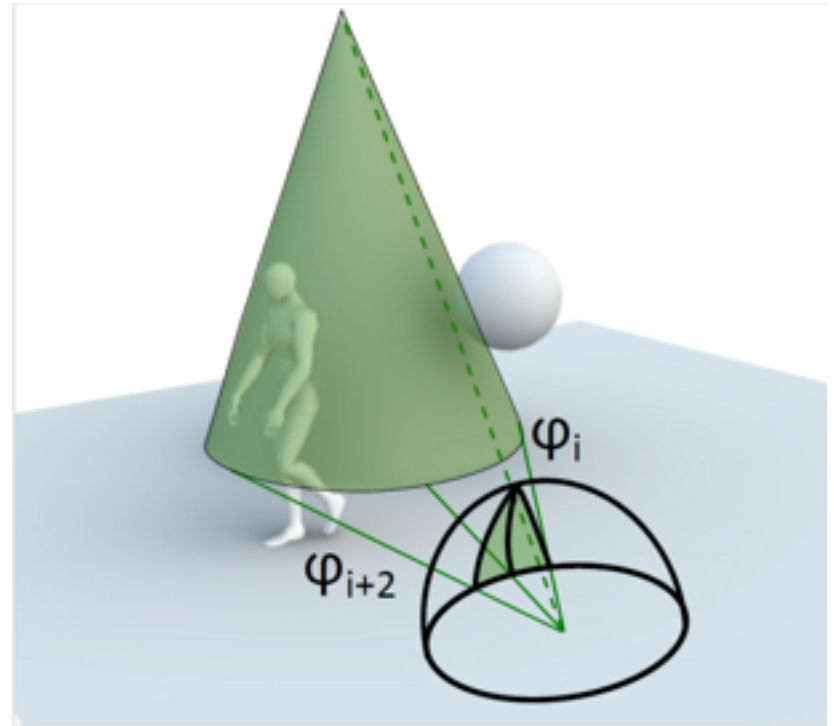
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$$\{ \mathbf{v}_{\log}^0, \mathbf{v}_{\log}^1, \dots, \mathbf{v}_{\log}^{B-1} \}$$



dynamic height field geometry

$$\mathbf{v}_{\log}^{\text{HF}}$$

- the **total** log-SH visibility vector is
$$\mathbf{V}_{\log} = \mathbf{v}_{\log}^{\text{HF}} + \sum_{b=0}^{B-1} \mathbf{v}_{\log}^b$$

SH Exponentiation [RWS*06]

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- Given any log-SH coefficient vector \mathbf{f}_{\log} , we use **SH exponentiation** to compute the (primal-domain) SH coefficients \mathbf{f}

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- A series expansion of the exponential, projected into SH

SH Exponentiation [RWS*06]

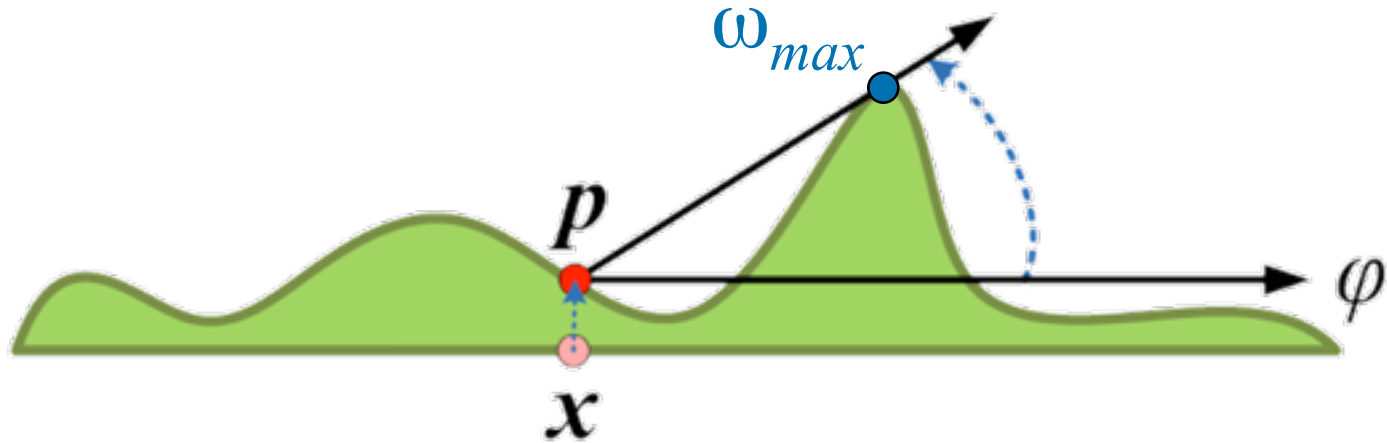
- Given any log-SH coefficient vector \mathbf{f}_{\log} , we use **SH exponentiation** to compute the (primal-domain) SH coefficients \mathbf{f}
- We use the **HYBrid** SH exponentiation method [RWS*06]
- A series expansion of the exponential, projected into SH
- Improved numerical stability with:
 - DC isolation
 - optimal linear-order approximation
 - SH scaling & squaring product accumulation

$$\mathbf{f} = \exp(\mathbf{f}_{\log}) \approx \mathbf{1} + \mathbf{f}_{\log} + \frac{\mathbf{f}_{\log}^2}{2} + \frac{\mathbf{f}_{\log}^3}{3!} + \dots$$

Summary of Main Ideas

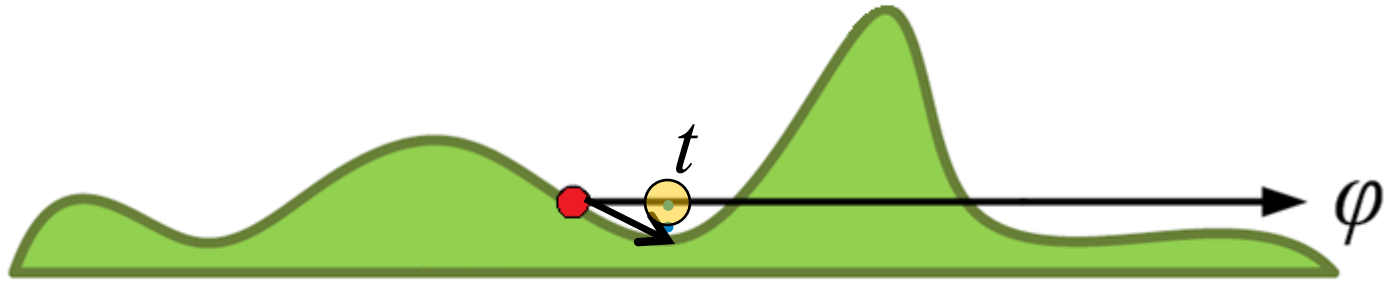
1. compute *HF self-visibility* (in *log-SH space*)
 - create multi-resolution height **pyramids**
 - sample from pyramid levels
 - pre-filter data
 - compose visibility **analytically** in log-space
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3. compute *mesh cast-visibility* (onto HF) **and self-visibility**
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HF Definitions and Notation [SN08]



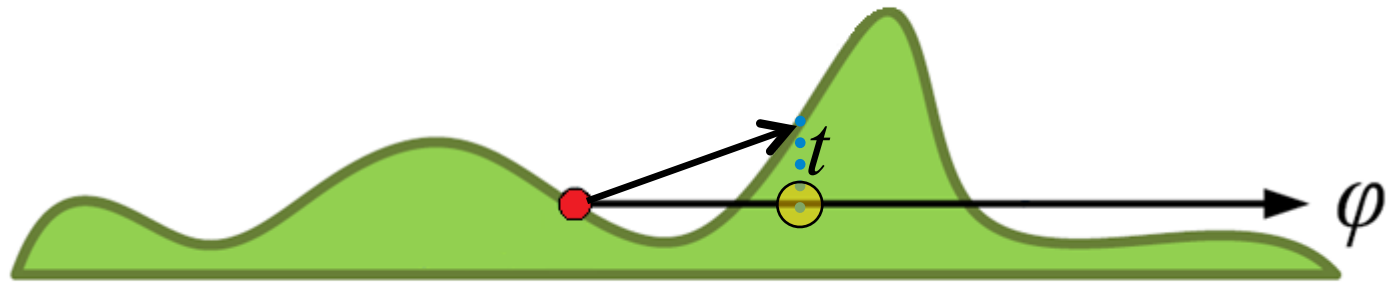
Need to find maximum blocking angle ω_{max} along direction φ .

Calculating the Max Blocking Angle



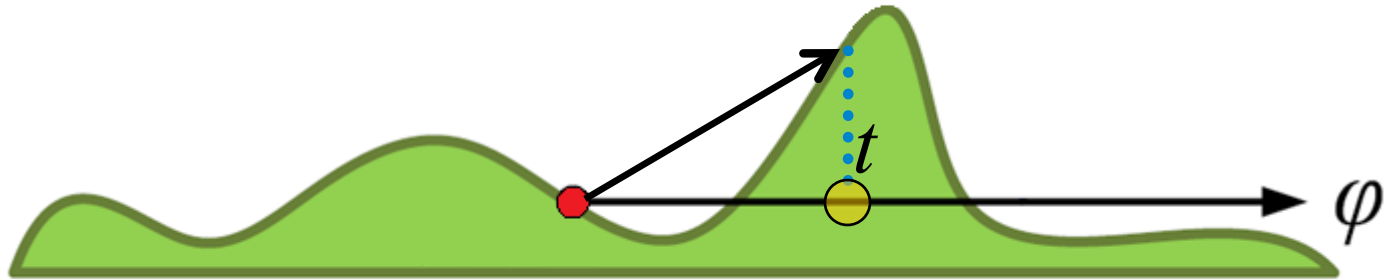
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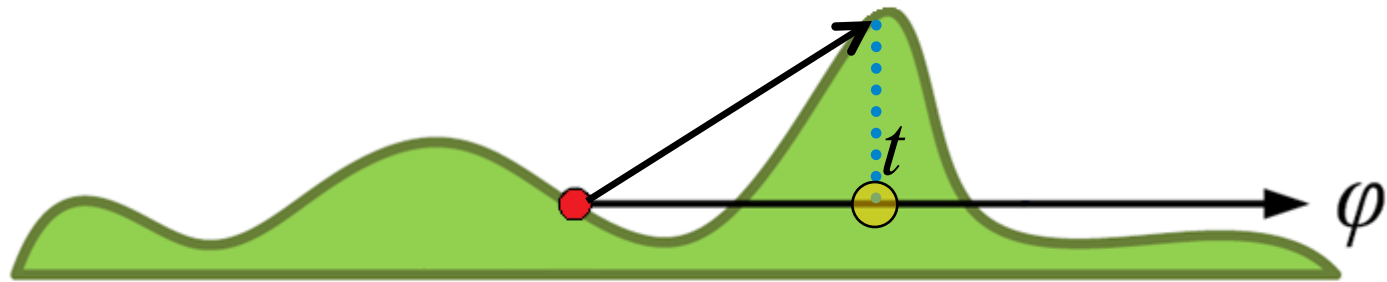
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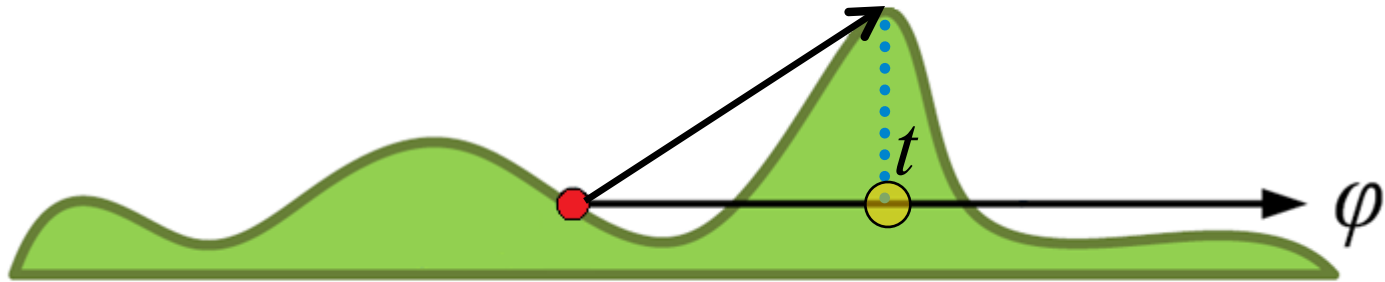
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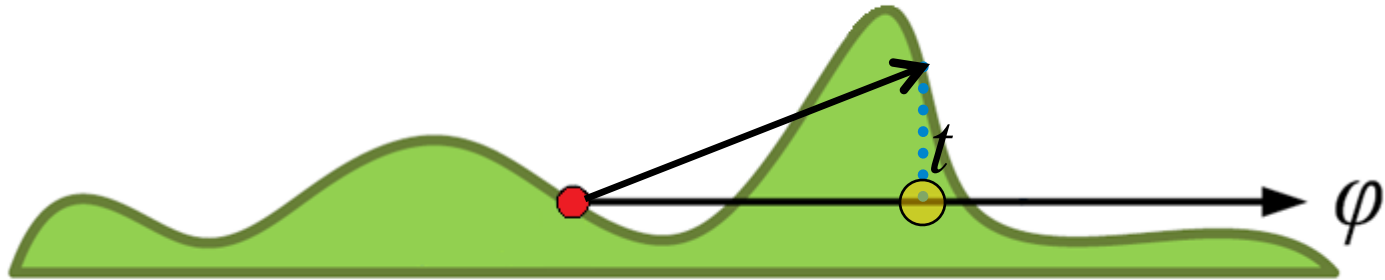
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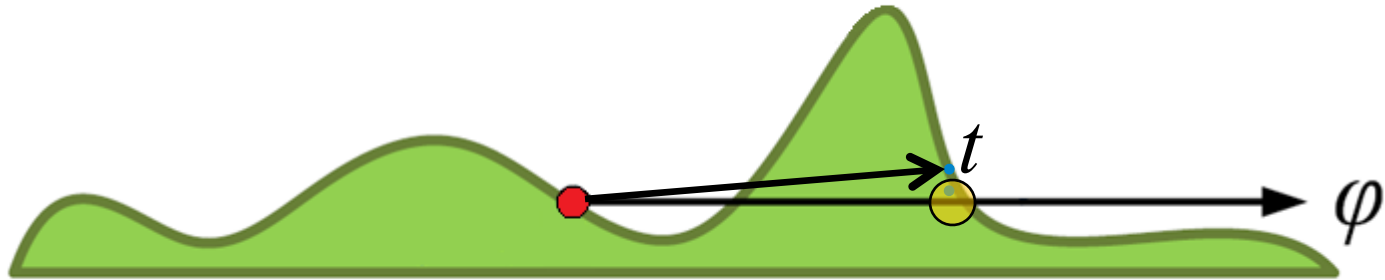
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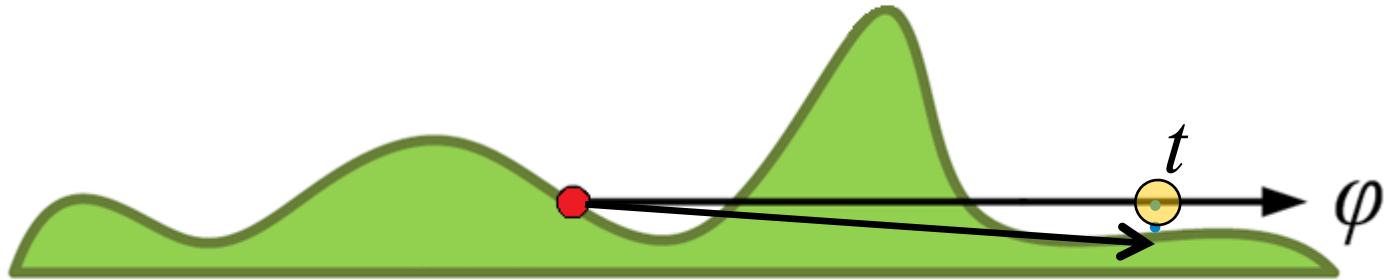
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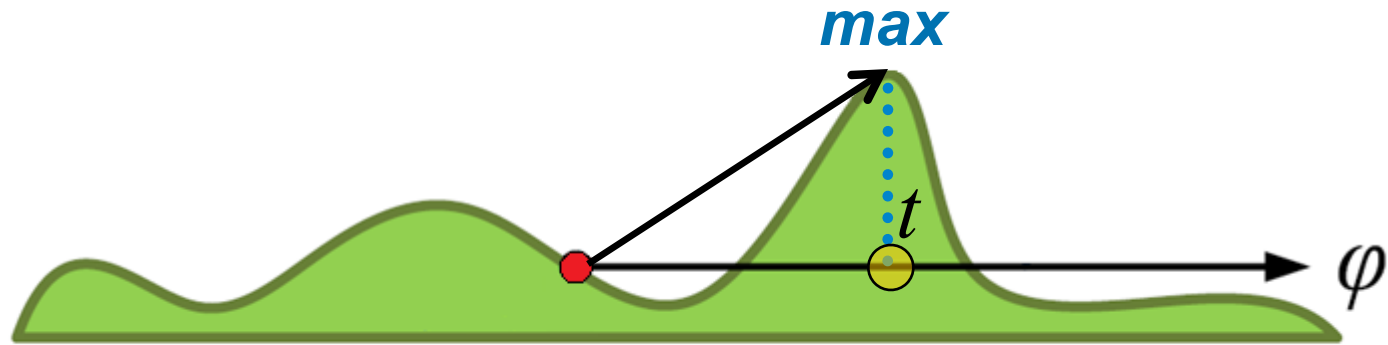
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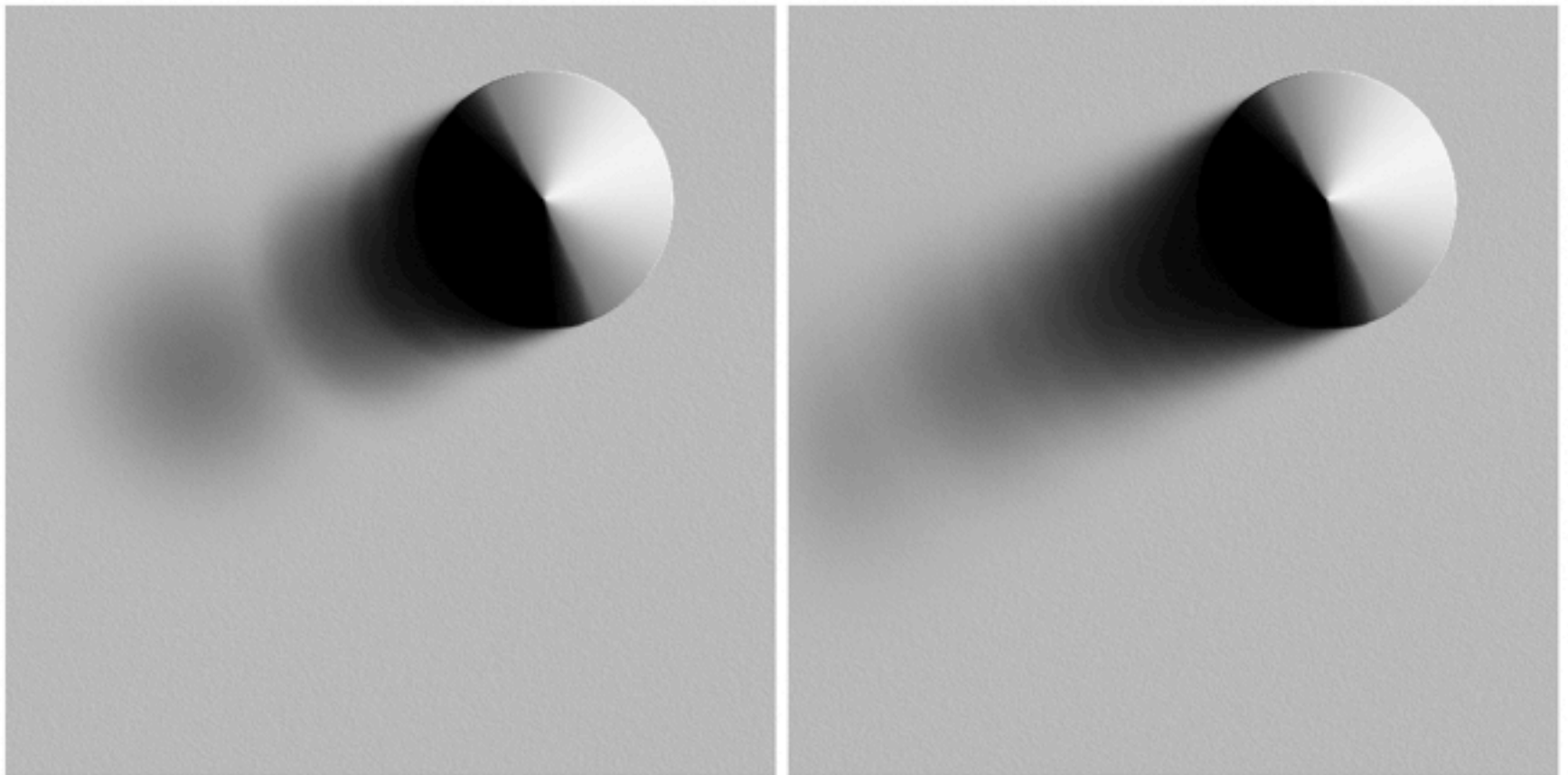
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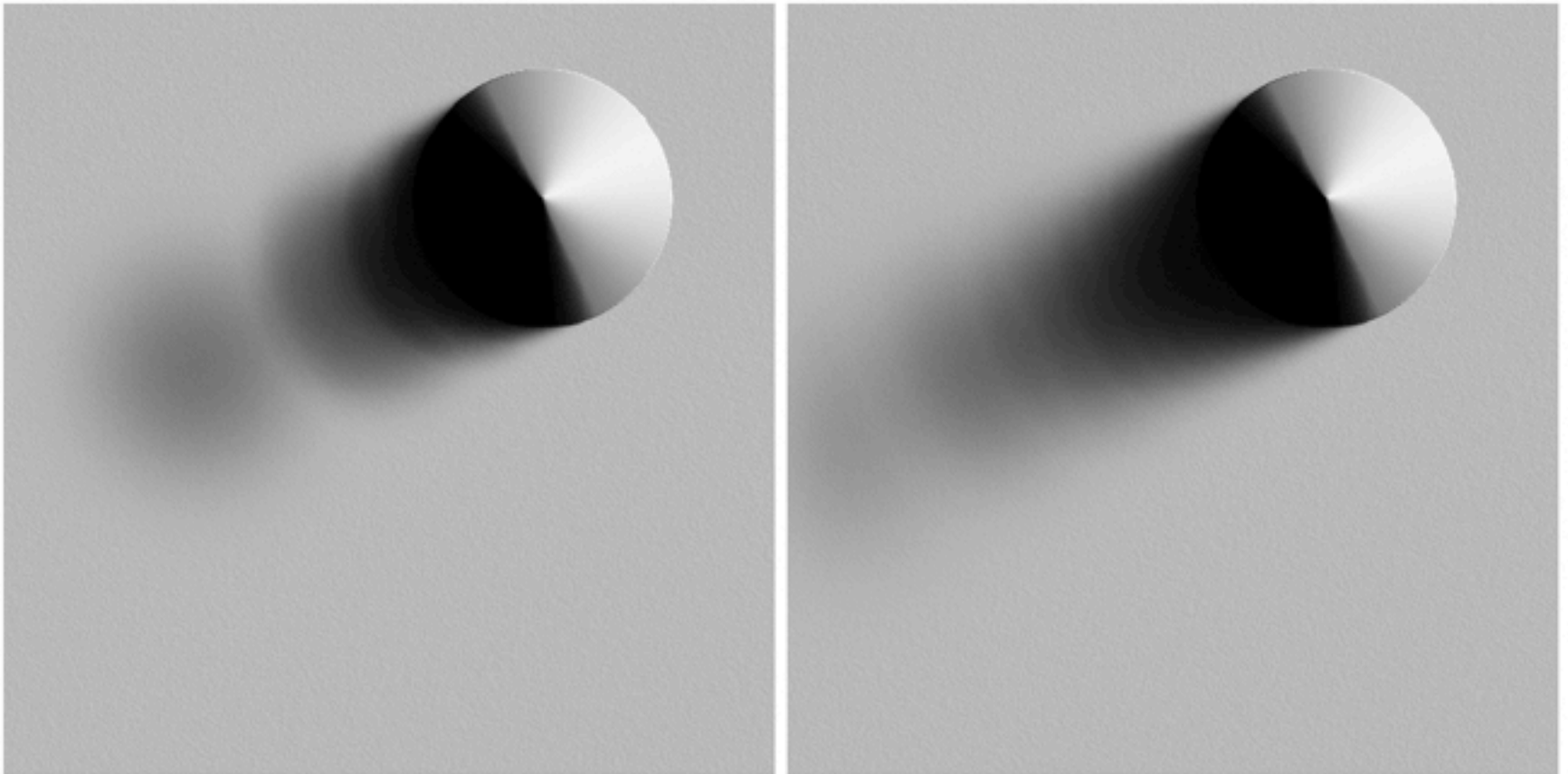


[SN08]

Brute Force Sampling [SN08]

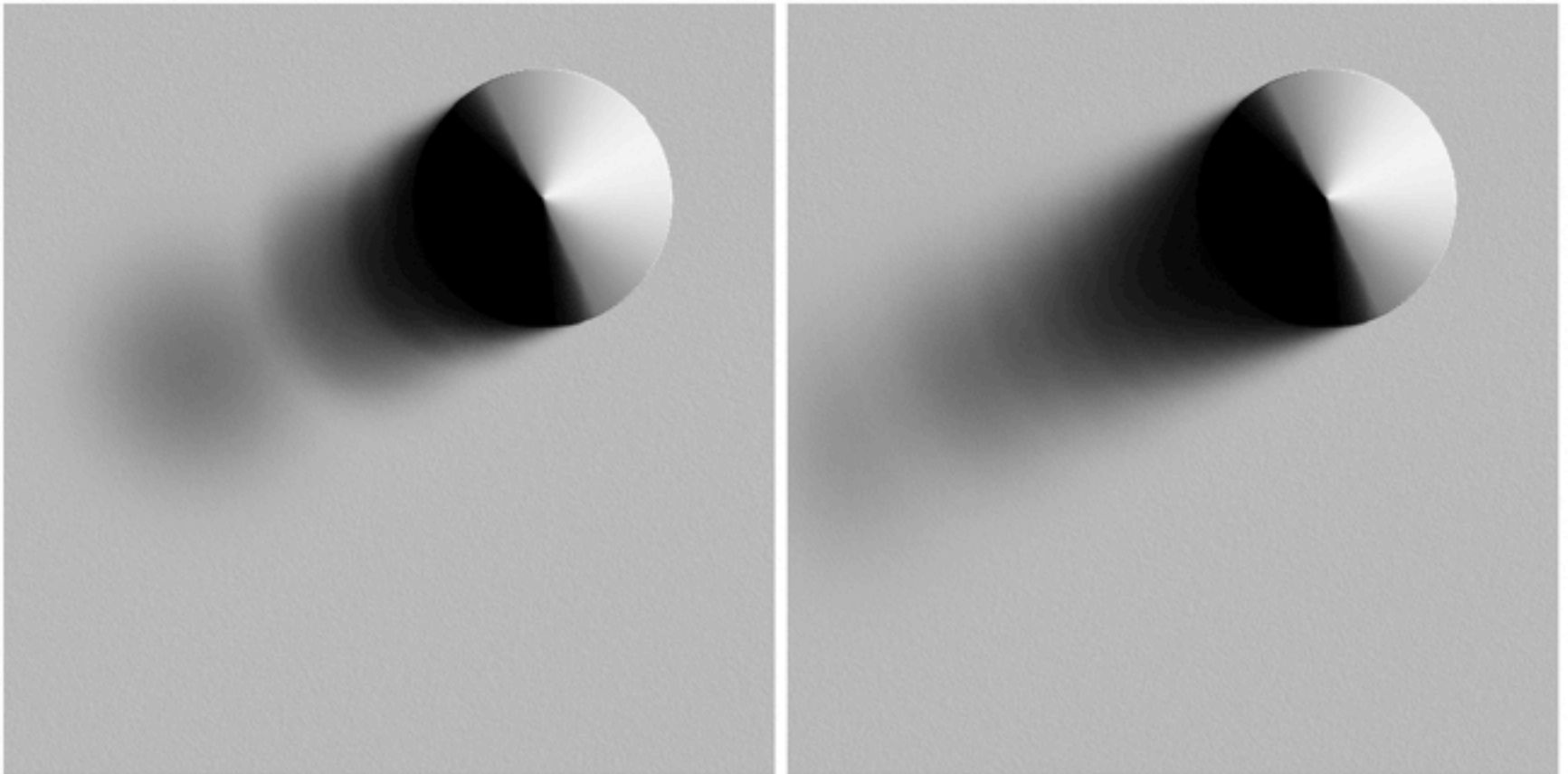


Brute Force Sampling [SN08]



Problem: *aliasing* – need *many* samples in t .

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Solution: *prefilter* data, apply *multi-scale sampling*.

Multi-Resolution Height Sampling [SN08]

f_i

height pyramid level i

$$\tau_i = 2^{f(i)}$$

sampling distance for level i

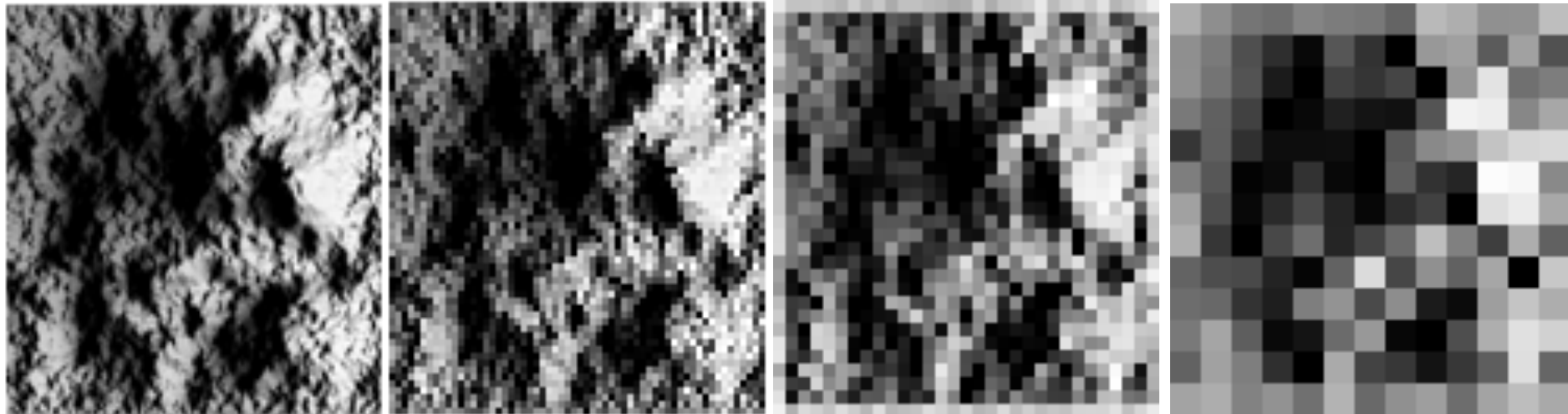
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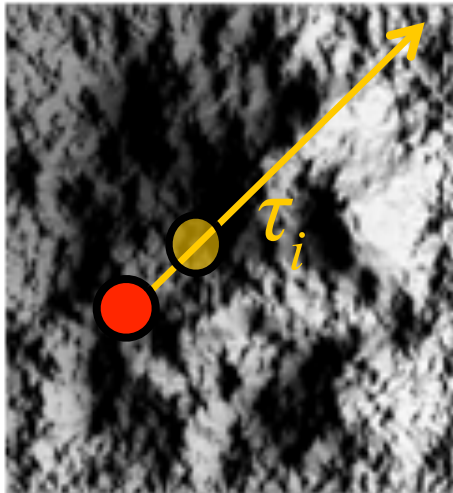
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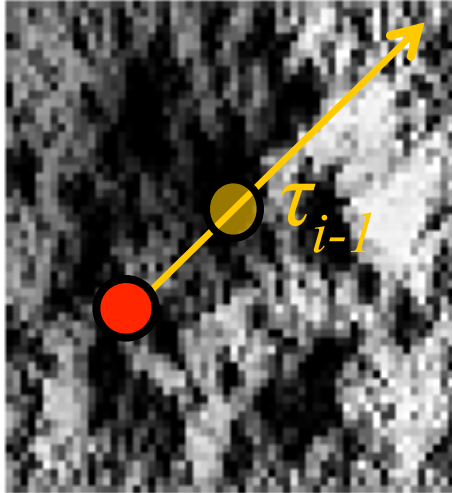
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Sample coarser levels further from x .

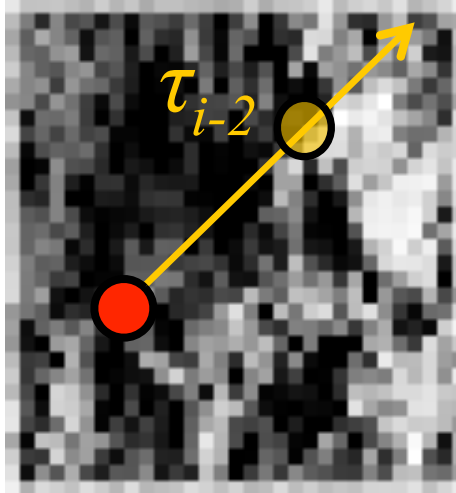
f_i



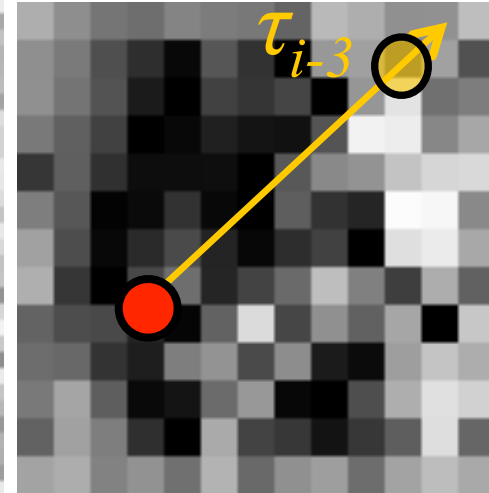
f_{i-1}



f_{i-2}



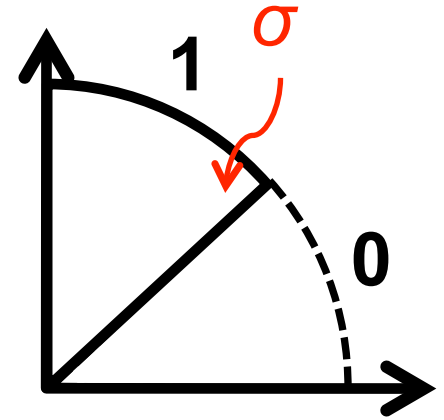
f_{i-3}



Elevation Visibility

- starting with binary *visibility* for an elevation slice:

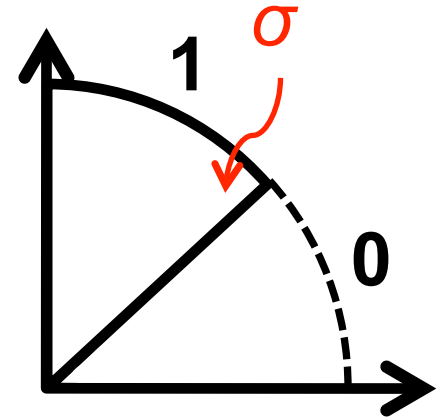
$$v(\omega; \sigma) = \begin{cases} 0, & \text{if } \omega \leq \sigma \\ 1, & \text{otherwise.} \end{cases}$$



Elevation Visibility

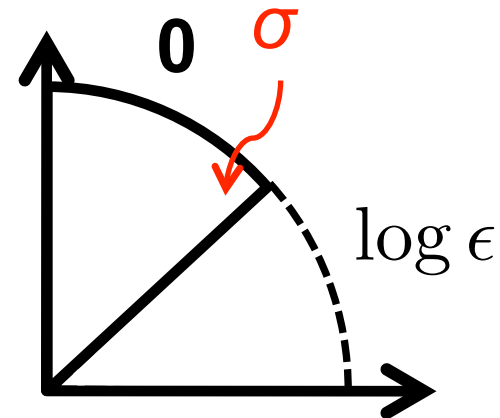
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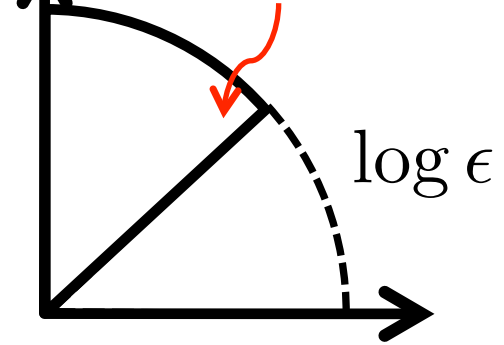


- we can express the *log-visibility* for the slice as

$$v_{\log}(\omega; \sigma) = \begin{cases} \log \epsilon, & \text{if } \omega \leq \sigma \\ 0, & \text{otherwise.} \end{cases}$$



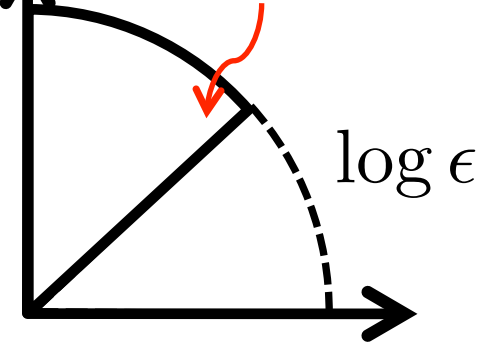
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and represent it analytically in the **Normalized Legendre Polynomial (NLP)** basis:

$$\mathbf{v}_{\log}(\sigma) = \int_{\pi/2 - \sigma}^{\pi} (\log \epsilon) \hat{\mathbf{P}}(\cos \theta) \sin \theta d\theta$$

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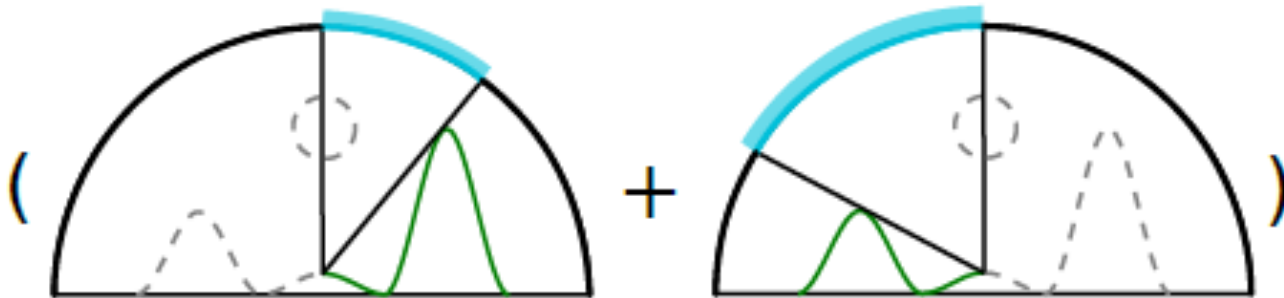
$$\begin{aligned} \mathbf{v}_{\log}(\sigma) &= \int_{\pi/2 - \sigma}^{\pi} (\log \epsilon) \hat{\mathbf{P}}(\cos \theta) \sin \theta d\theta \\ &= \log \epsilon \times \left[\frac{\sin \sigma + 1}{\sqrt{2}}, \frac{-3 \cos^2 \sigma}{2\sqrt{6}}, \frac{-5 \sin \sigma \cos^2 \sigma}{2\sqrt{10}}, \right. \\ &\quad \left. \frac{7 \cos^2 \sigma (-4 + 5 \cos^2 \sigma)}{8\sqrt{14}} \right] \end{aligned}$$

Accumulating HF Visibility

- in the primal domain: can **sum** SH visibility for each slice

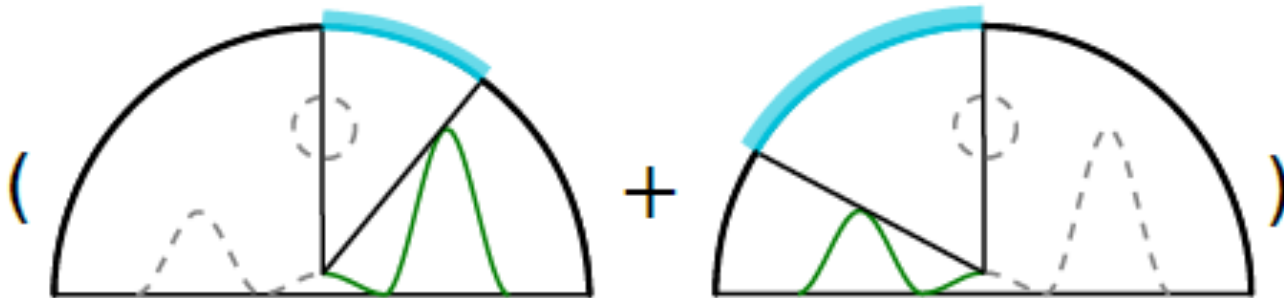
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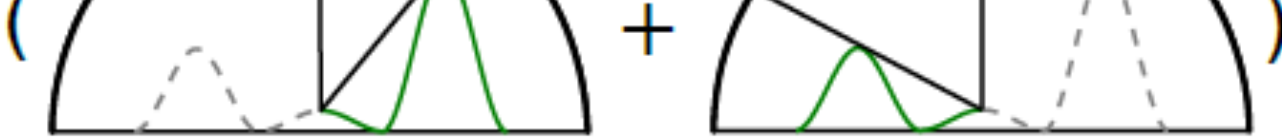


Accumulating HF Visibility

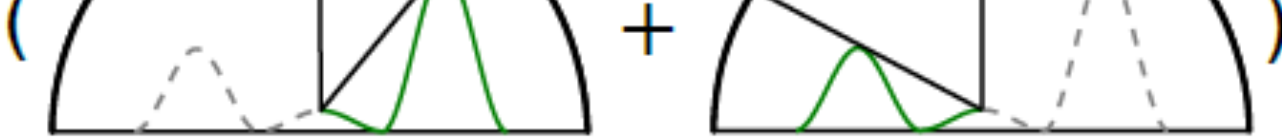
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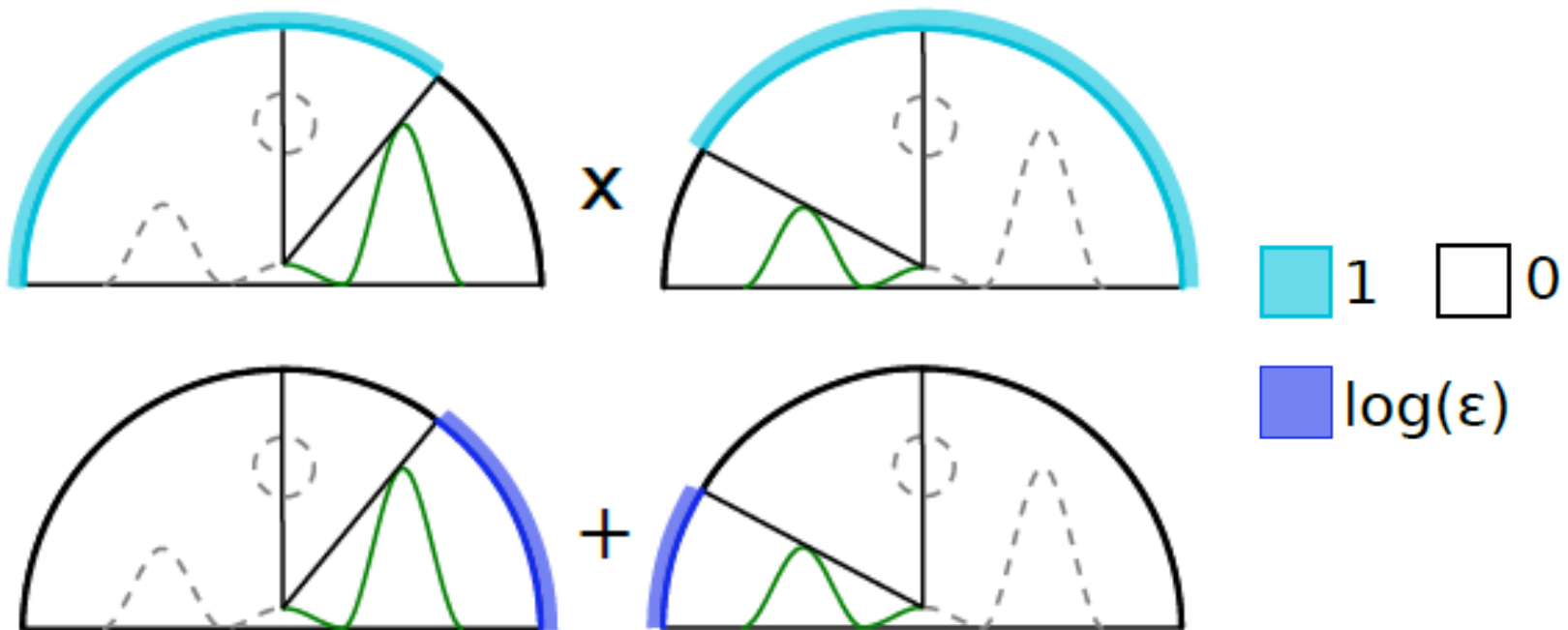
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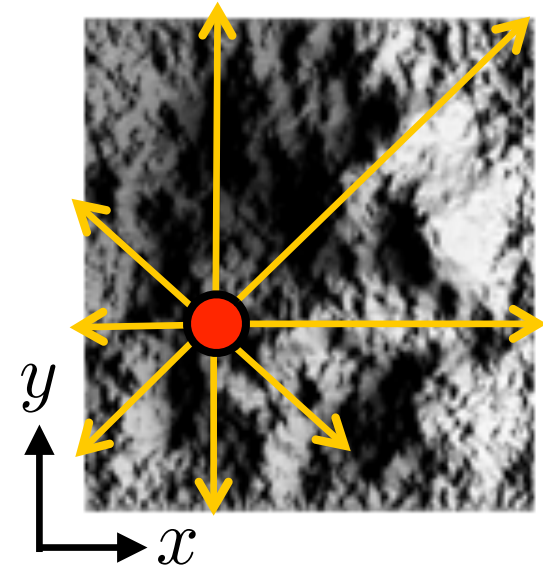


Visibility Slice Interpolation [NS09]

- already computed log-ZH azimuthal visibility, per-direction

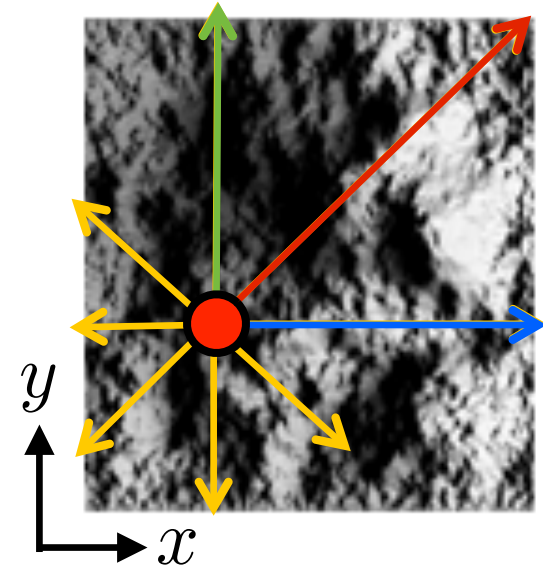
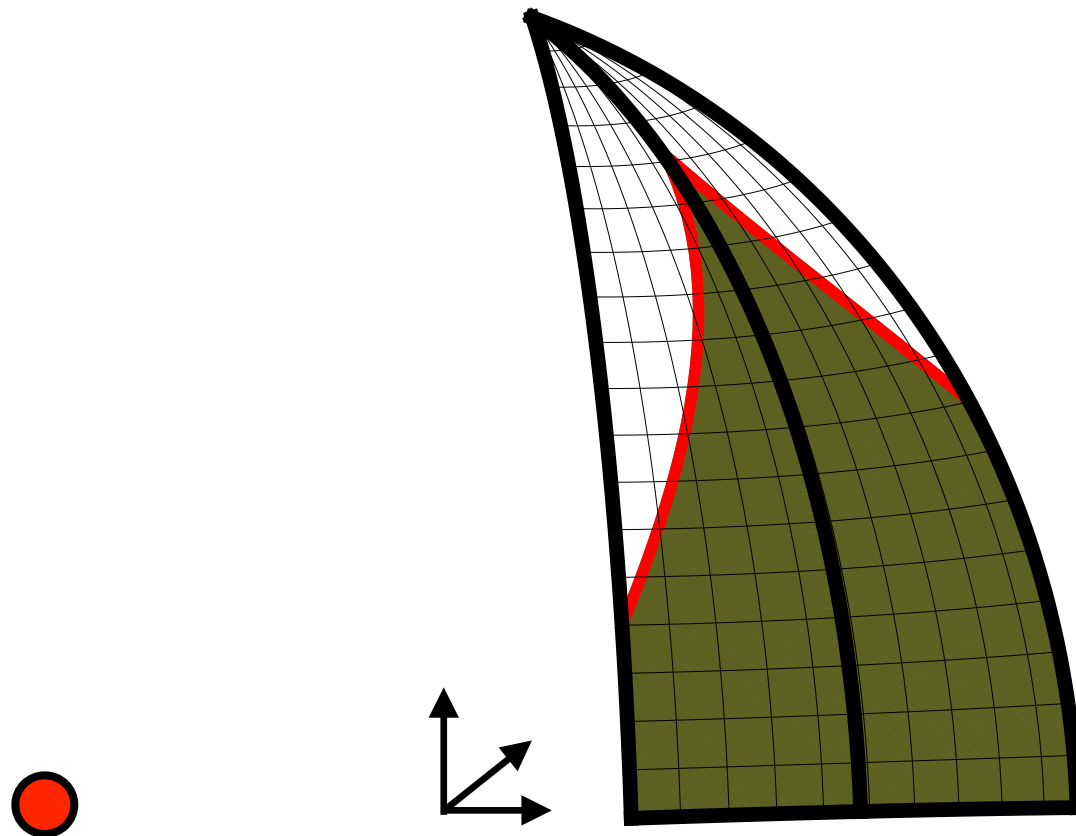
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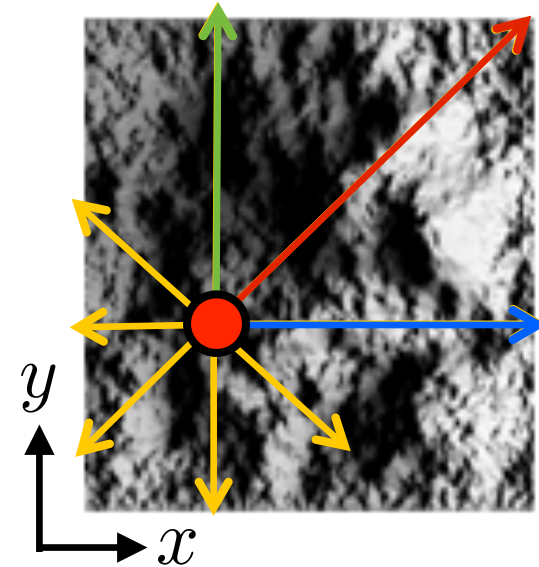
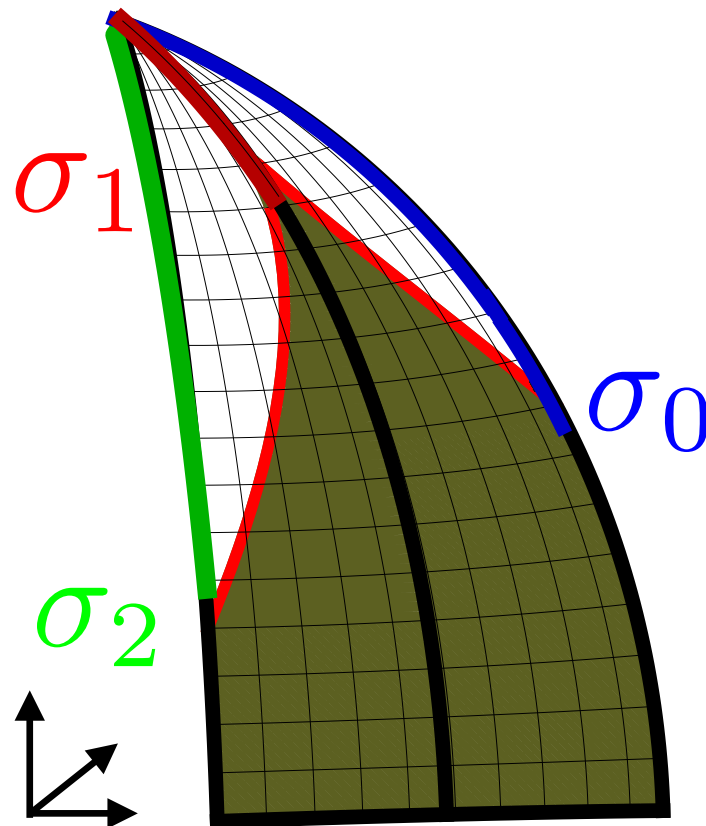
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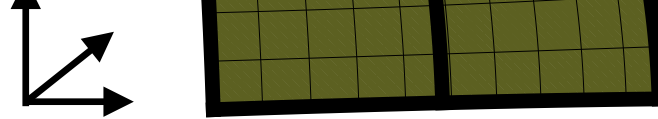
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Visibility Slice Interpolation [NS09]

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- requires 1 precomputed interpolation + projection matrix

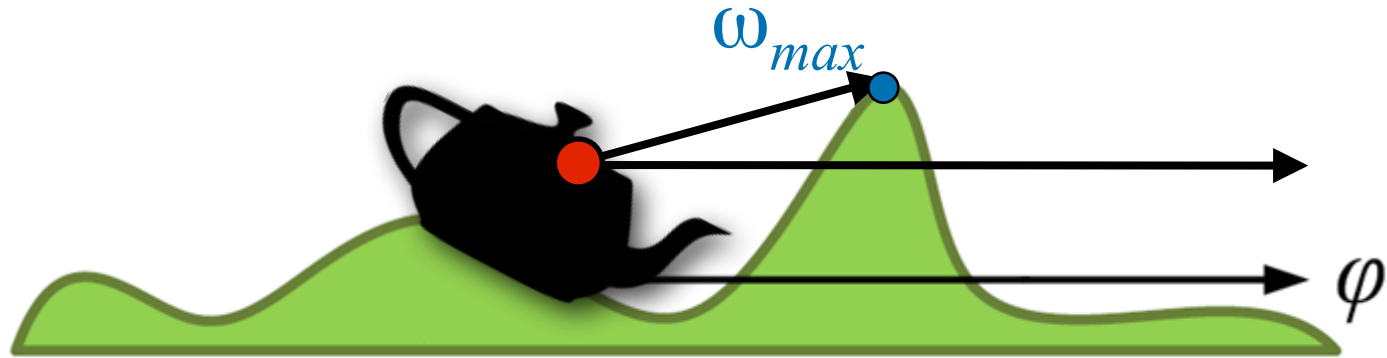
$$\mathbf{v}_{\log}^{\text{wedge}} = \begin{bmatrix} \mathbf{M}_{lin} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\log}(\sigma_0) \\ \mathbf{v}_{\log}(\sigma_1) \end{bmatrix}$$

- **rotate** and **sum** across each wedge's $\mathbf{v}_{\log}^{\text{wedge}}$ to form final log-SH vector $\mathbf{v}_{\log}^{\text{HF}}$

Summary of Main Ideas

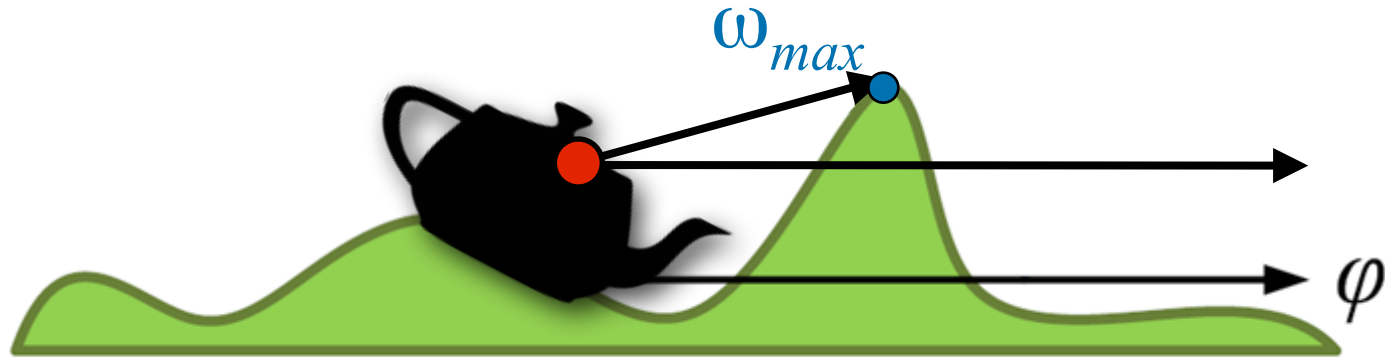
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Height Field Cast Visibility onto Meshes



Need to find ω_{max} on mesh **shading point** along each direction φ

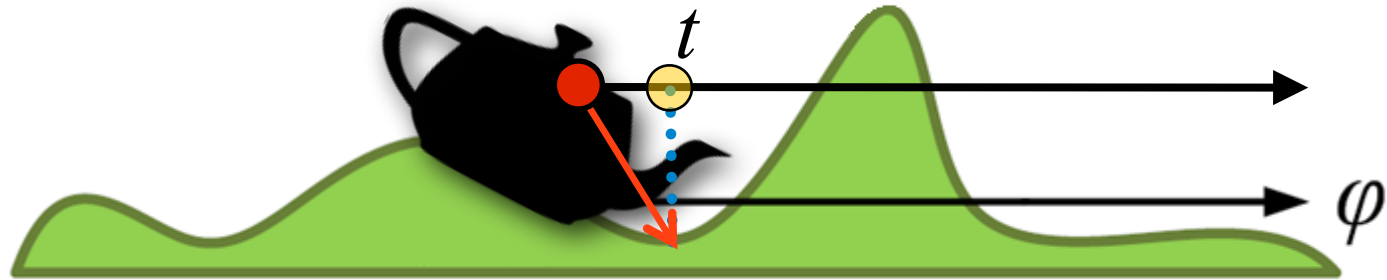
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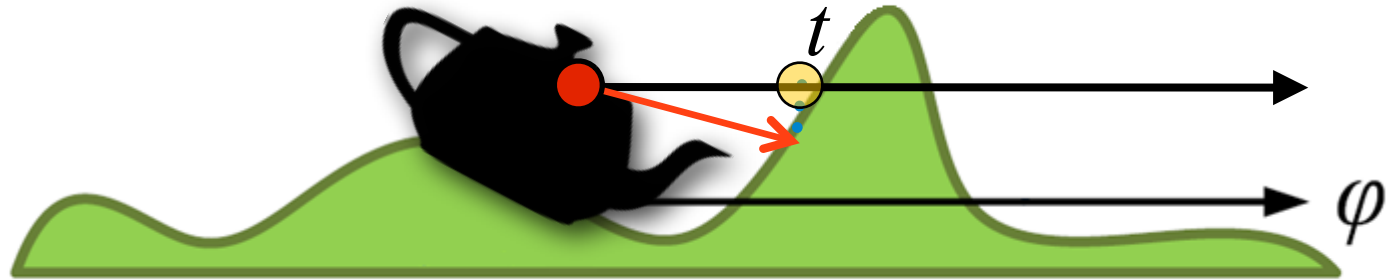
Need to find ω_{max} on **mesh shading point** along each direction φ

- Assume an infinite plane for the HF base elevation
- minimum blocking angle can't go negative

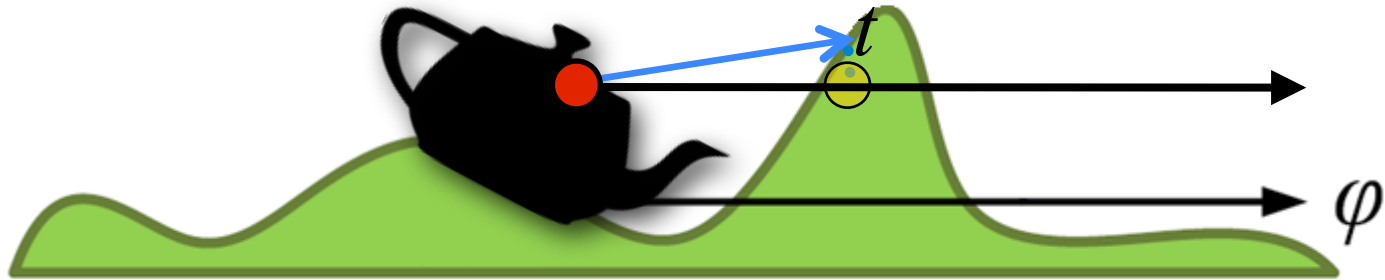
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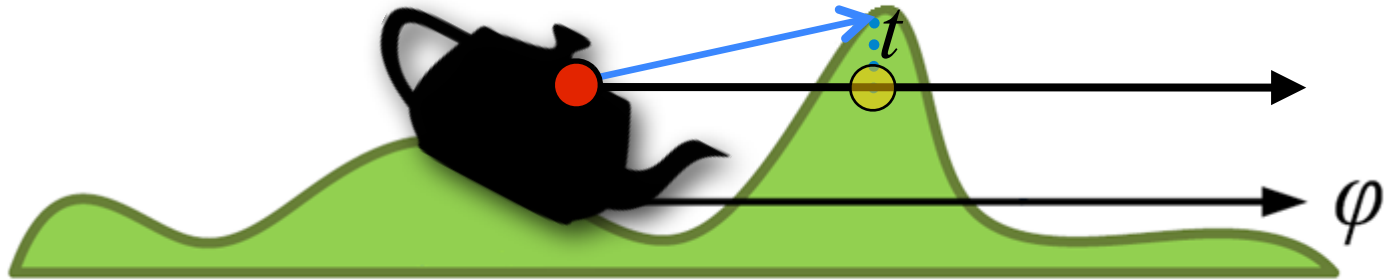
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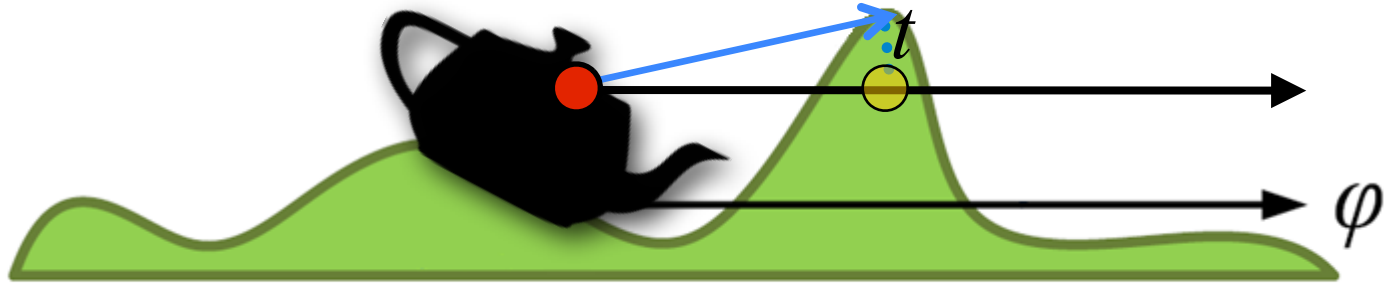
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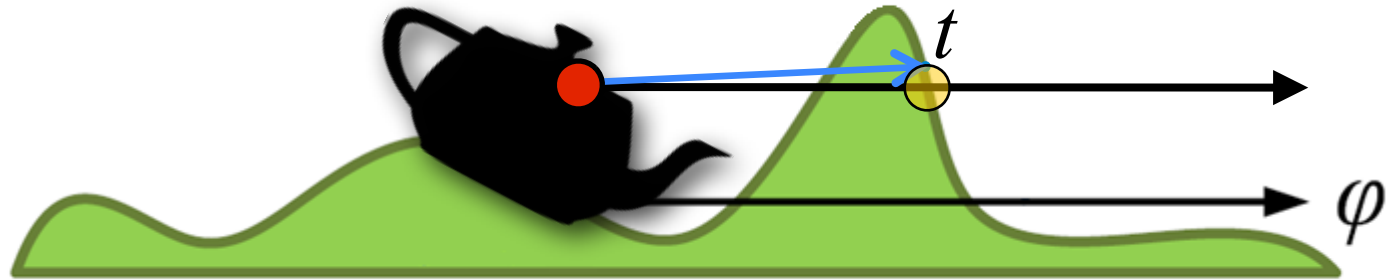
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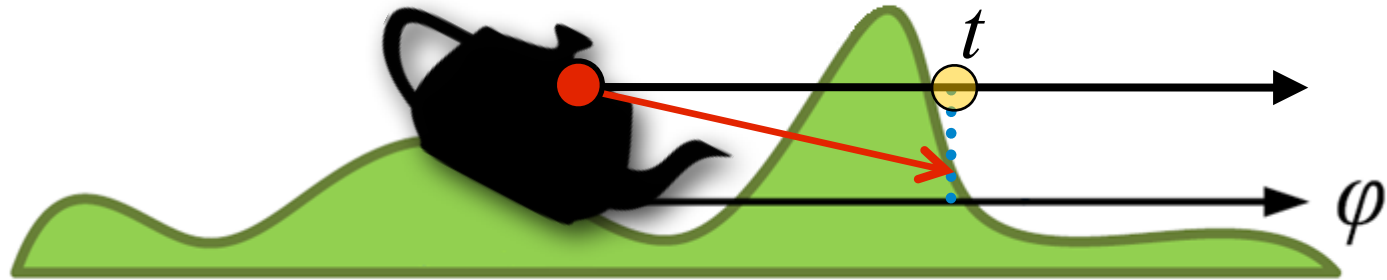
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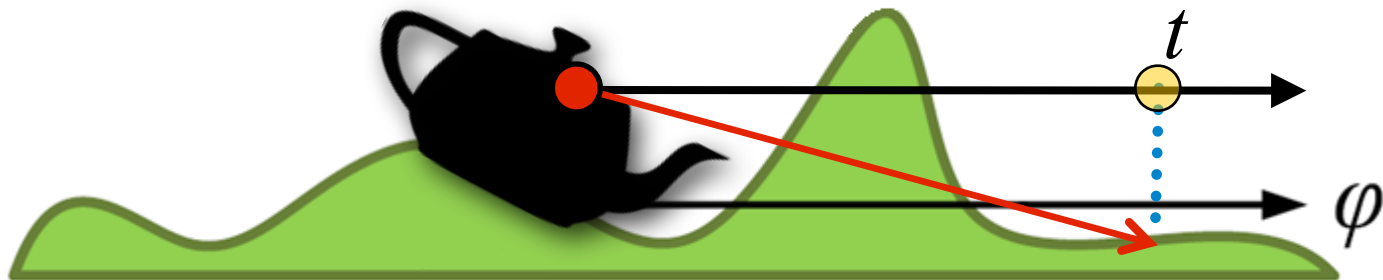
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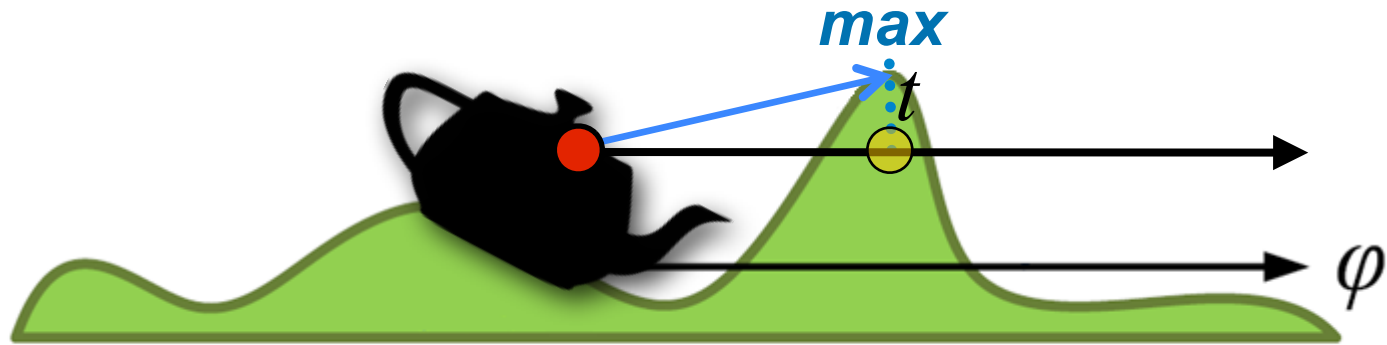
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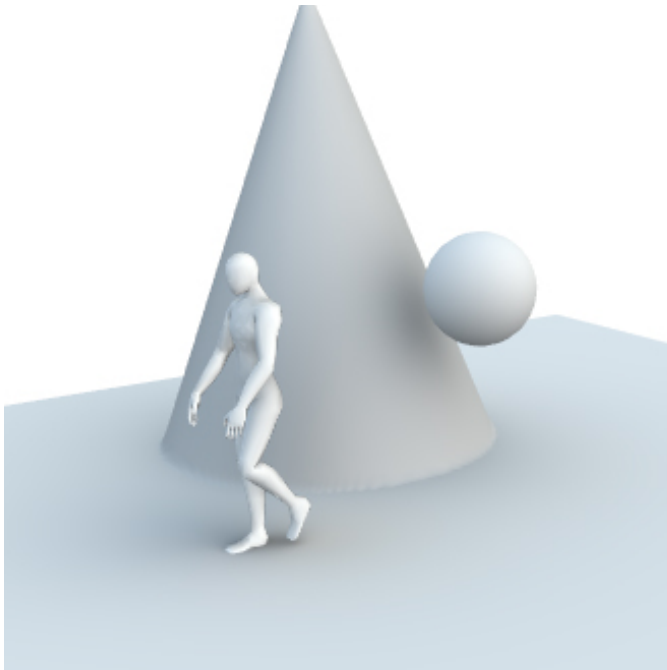


Summary of Main Ideas

1. compute *HF self-visibility* (in *log-SH space*)
2. compute *HF cast-visibility* (onto meshes)
3. compute *mesh cast-visibility* (onto HF) **and self-visibility**
 - extend traditional SH exponentiation approach [**RWS*06;SGNS07**]
 - **decompose** dynamic mesh blockers into spheres
 - **compute & accumulate log-SH visibility** for spherical blockers
 - on the mesh shading points
 - **repeat** over the HF shading points
 - intelligently cull the sphere set during accumulation
 - reduces numerical accumulation error
4. accumulate total spherical visibility
5. compute log-SH BRDF and perform final shading

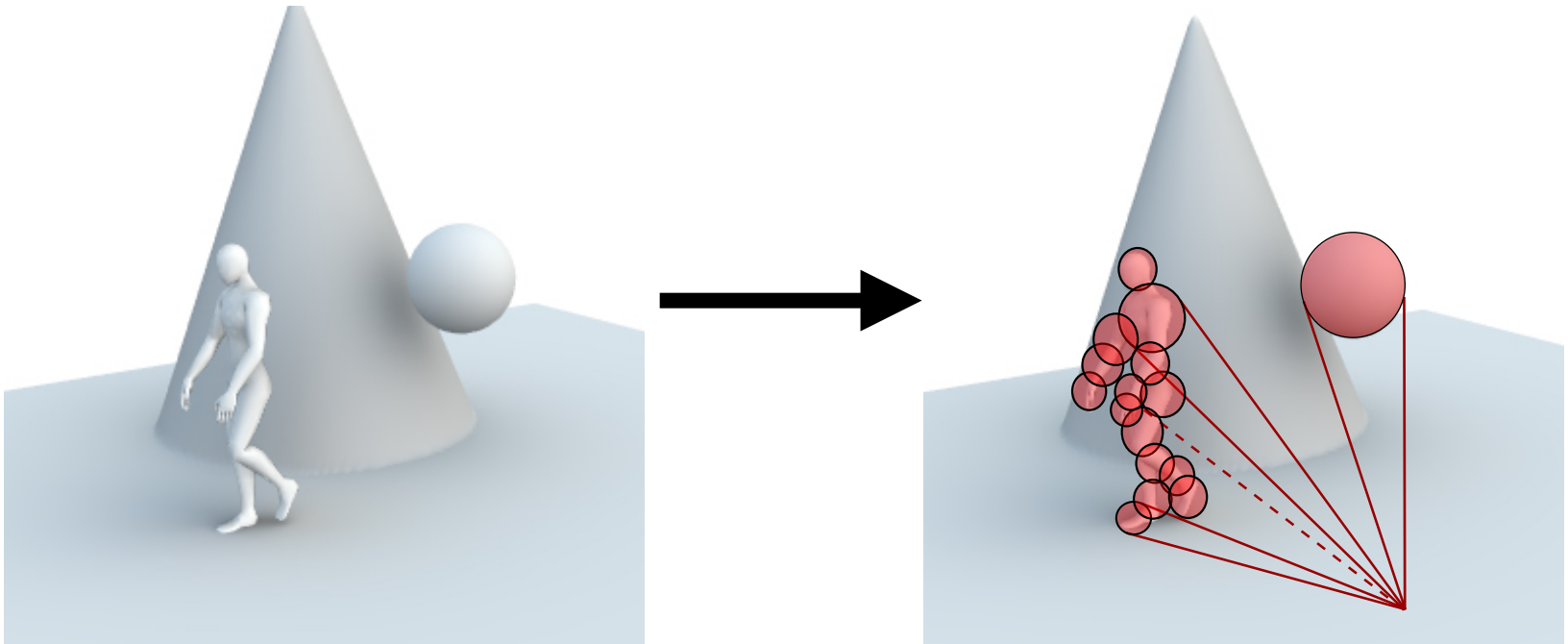
Spherical Blockers [RWS*06]

- approximate dynamic meshes with a set of spheres
 - precomputed once
 - skinned dynamically during animation/deformation



Spherical Blockers [RWS*06]

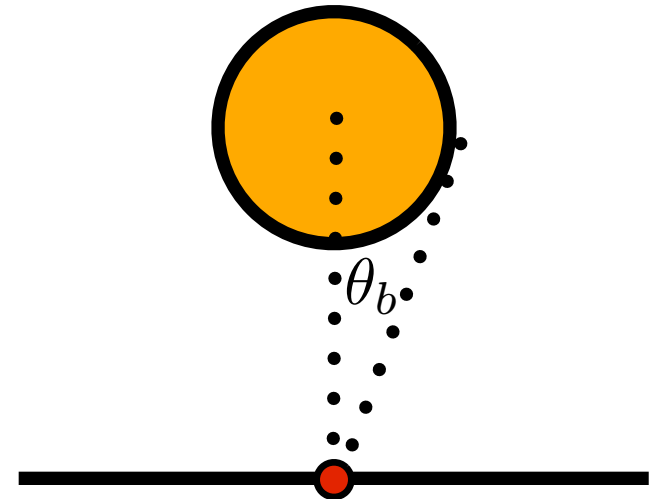
- approximate dynamic meshes with a set of spheres
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Spherical Blocker Log SH Visibility

- can compute log-visibility SH coefficients *analytically*
- begin with a canonical alignment:

$$\theta_b = \arcsin(r/d)$$

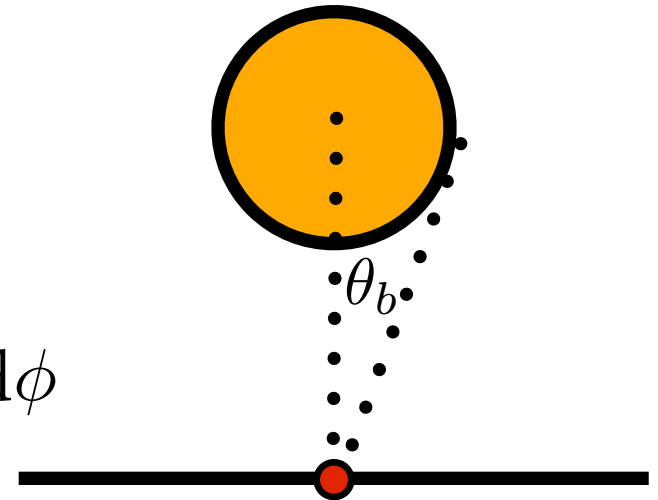


Spherical Blocker Log SH Visibility

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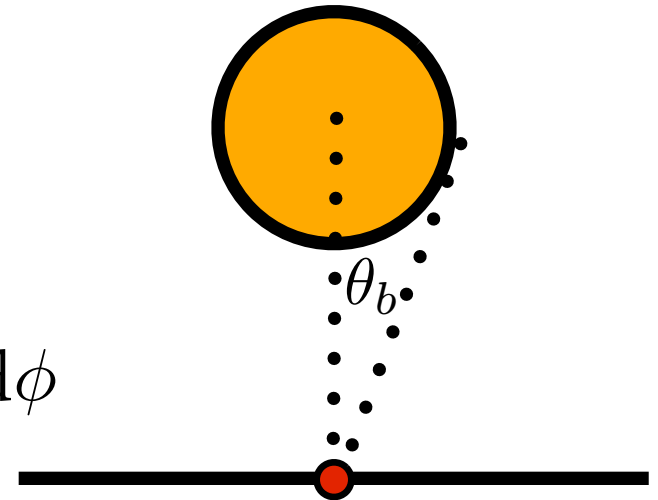


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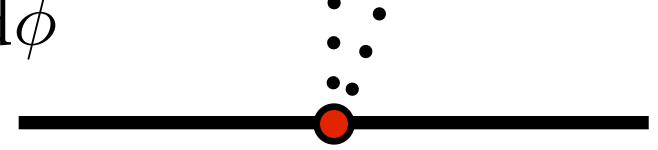
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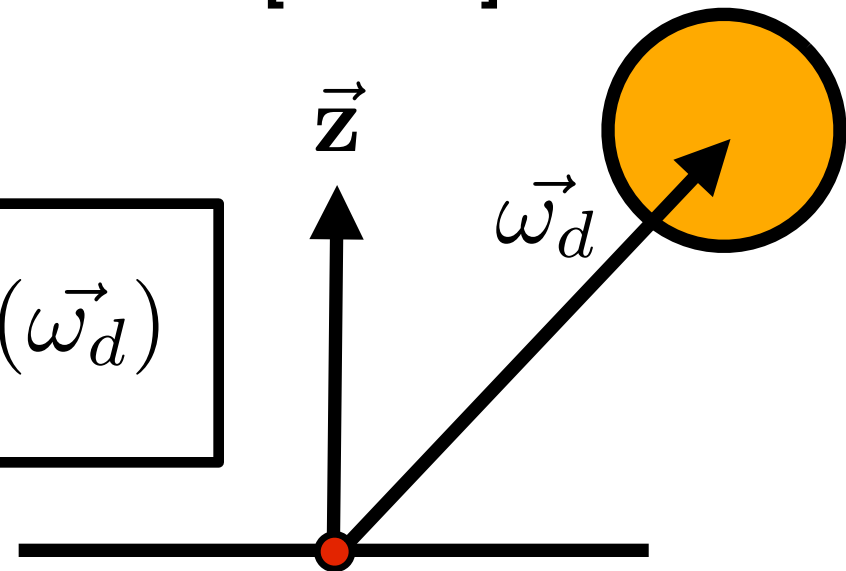


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- align to shading frame with ZH rotation **[SLS05]**

$$\mathbf{v}_{l,m}^{\log} = \sqrt{\frac{4\pi}{2l+1}} \mathbf{v}_l^{\log} y_l^m(\vec{\omega}_d)$$



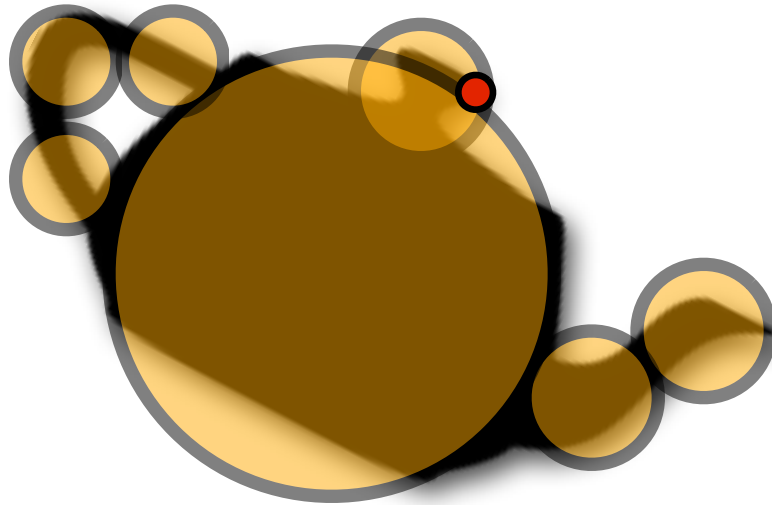
Spherical Blocker Self- & Cast- Shadows

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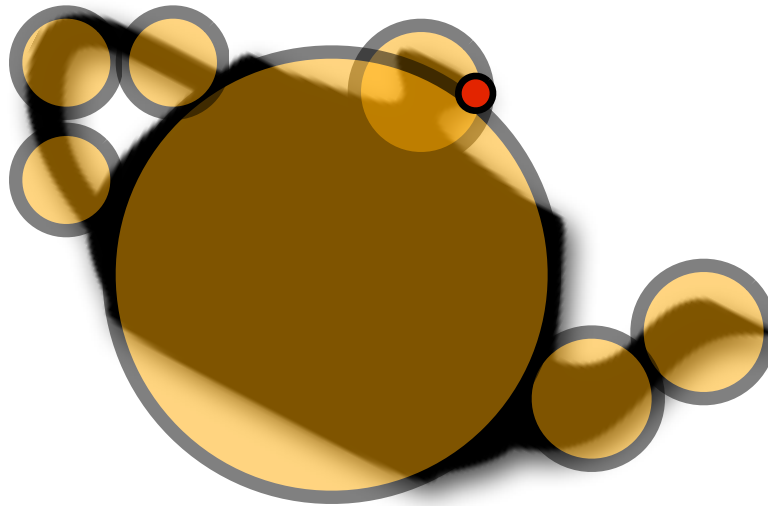
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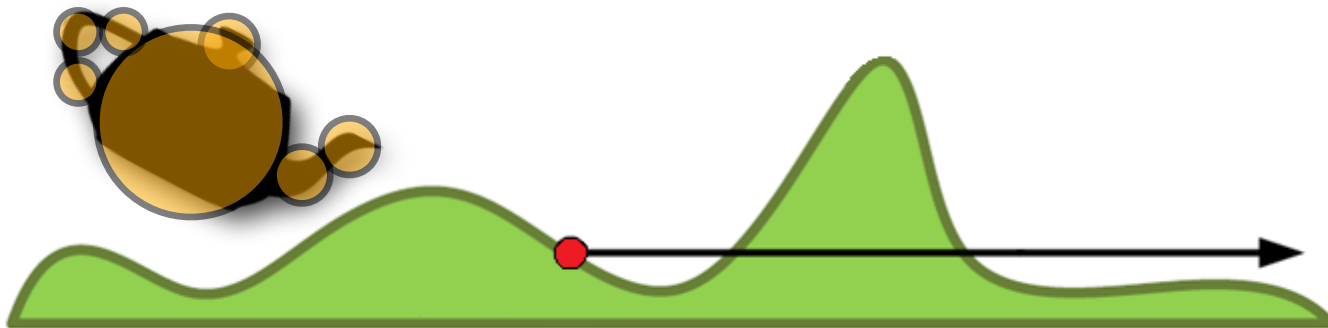


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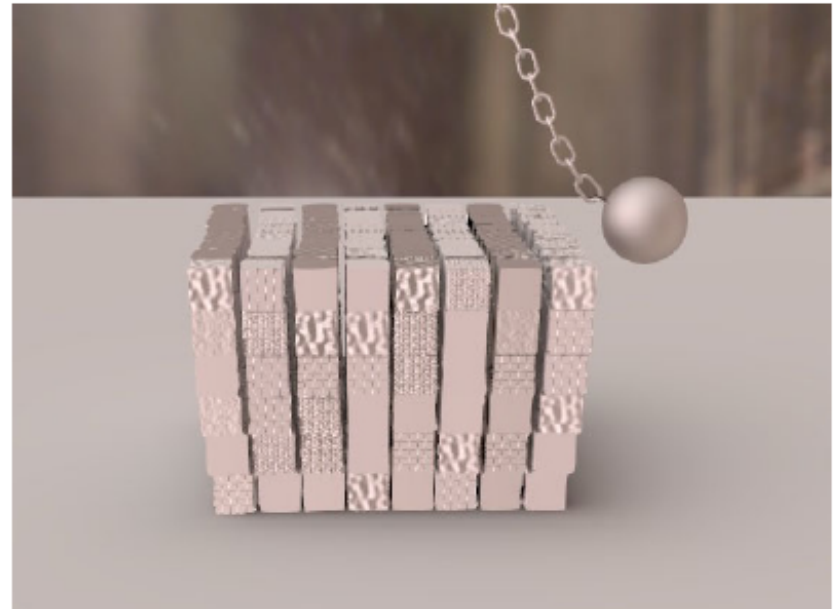
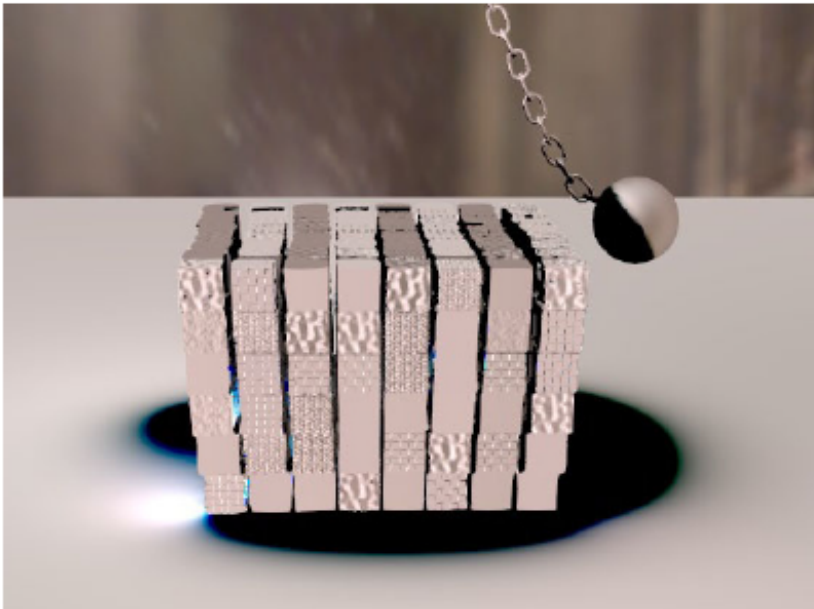
- **and** dynamic object *cast-occlusion* onto the HF



Ratio Attenuation

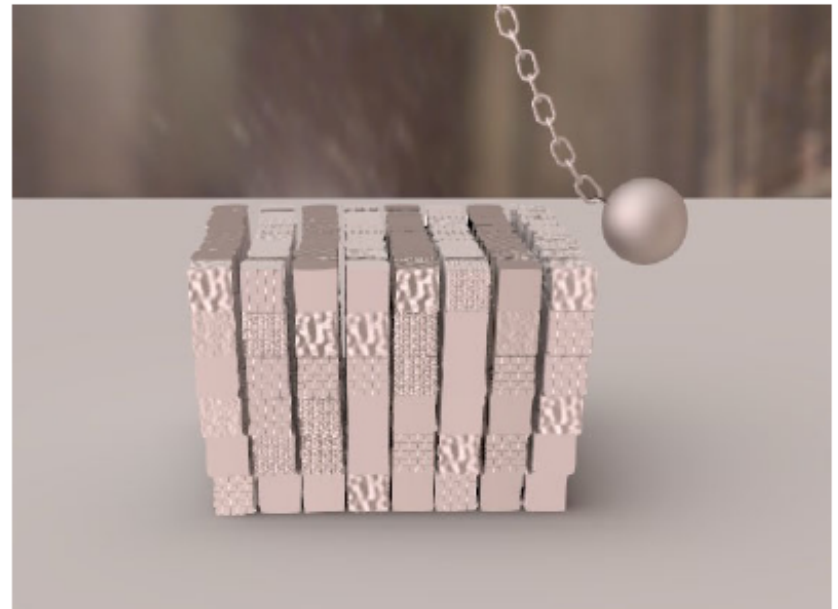
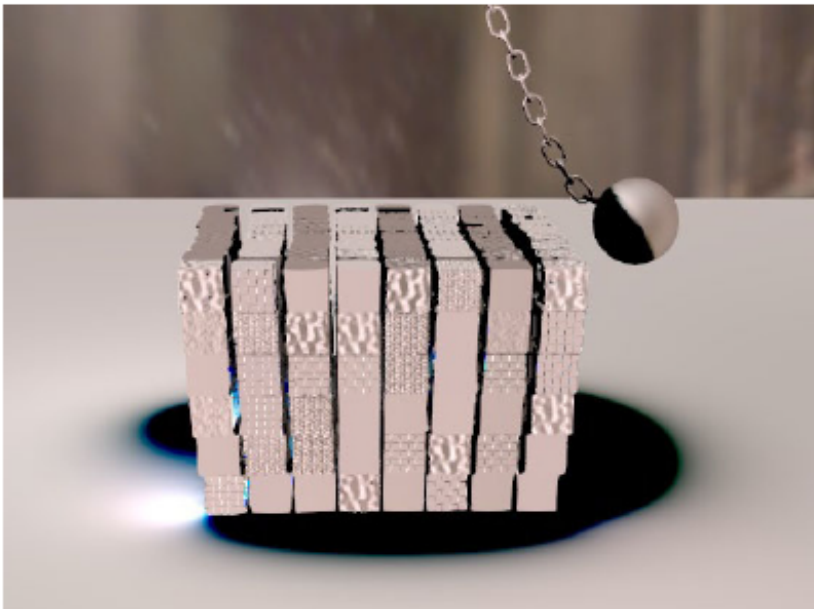
Ratio Attenuation

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Ratio Attenuation

- SH exponentiation suffers from accumulation error when there are **many overlapping** blocker spheres
- we reduce accumulation error by:
 - weighting log-SH visibility by blocker solid angle, and
 - only accumulating blockers in upper shading hemisphere

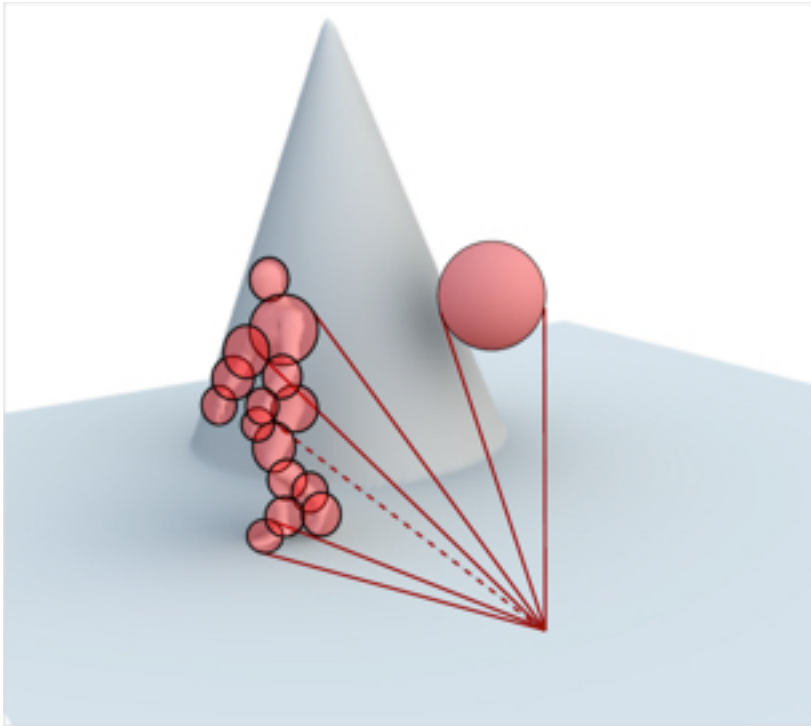


Summary of Main Ideas

1. compute *HF self-visibility* (in *log-SH space*)
2. compute *HF cast-visibility* (onto meshes)
3. compute *mesh cast-visibility* (onto HF) **and self-visibility**
4. accumulate total spherical visibility
 - combine **per-slice** HF (log) visibility to form **full** spherical visibility [**NS09**]
 - **accumulate** dynamic mesh blocker log-visibility and HF log-visibility
 - perform **SH exponentiation**
5. compute log-SH BRDF and perform final shading

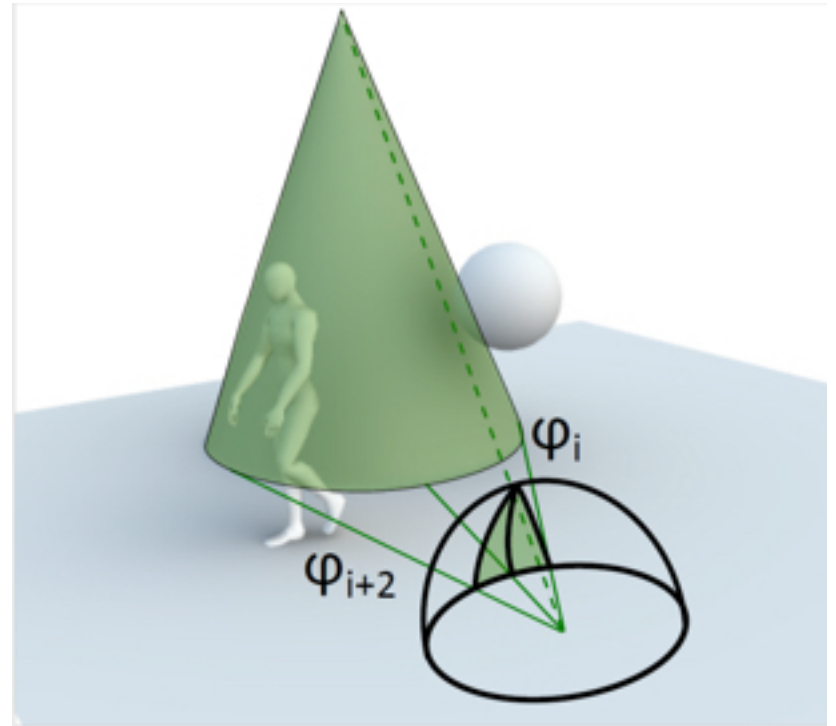
Accumulate Log-SH Visibility

Given spherical log-SH visibility for



dynamic blocker “meshes”

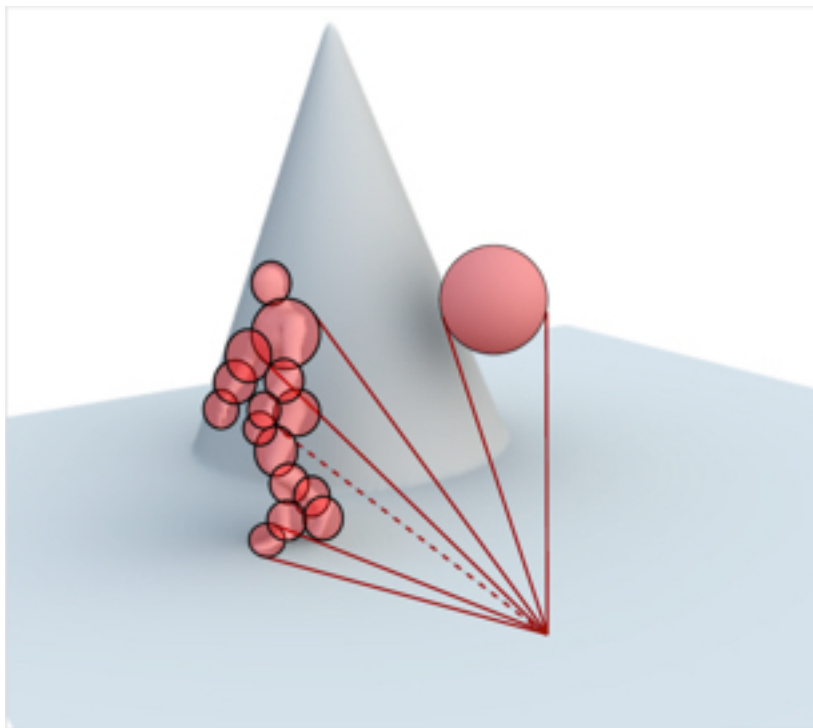
$$\{ \mathbf{v}_{\log}^0, \mathbf{v}_{\log}^1, \dots, \mathbf{v}_{\log}^{B-1} \}$$



dynamic height field geometry

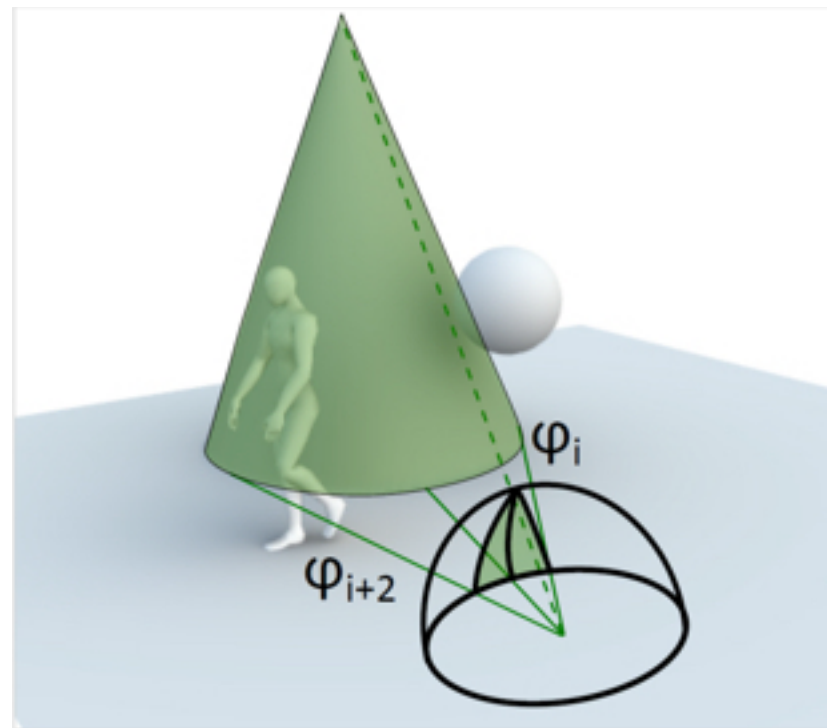
$$\mathbf{v}_{\log}^{\text{HF}}$$

- the **total** log-SH visibility vector is
$$\mathbf{V}_{\log} = \mathbf{v}_{\log}^{\text{HF}} + \sum_{b=0}^{B-1} \mathbf{v}_{\log}^b$$



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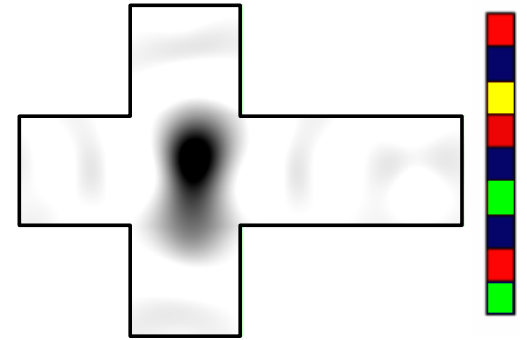
$$\mathbf{V} = \exp(\mathbf{V}_{\log}) \approx \mathbf{1} + \mathbf{V}_{\log} + \frac{\mathbf{V}_{\log}^2}{2} + \frac{\mathbf{V}_{\log}^3}{3!} + \dots$$

Summary of Main Ideas

1. compute *HF self-visibility* (in *log-SH space*)
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5. compute log-SH BRDF and perform final shading
 - simplify triple-product shading to double-product shading
 - **formulate** view-evaluated BRDF in log-SH space
 - **accumulate** BRDF with multi-product visibility

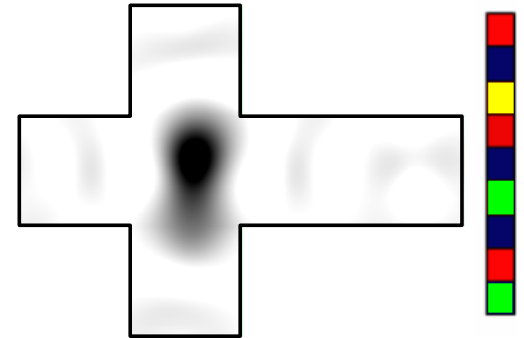
Traditional Triple Product SH Shading

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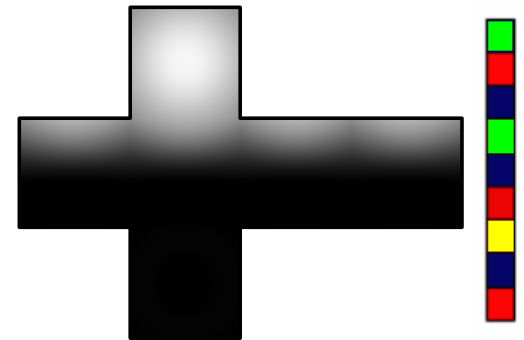


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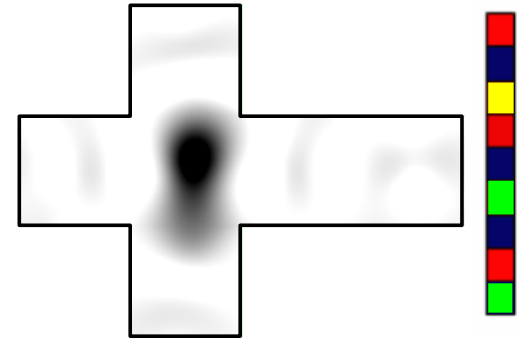


$$\mathbf{f}_r(\omega_o) \quad \text{view-evaluated BRDF}$$

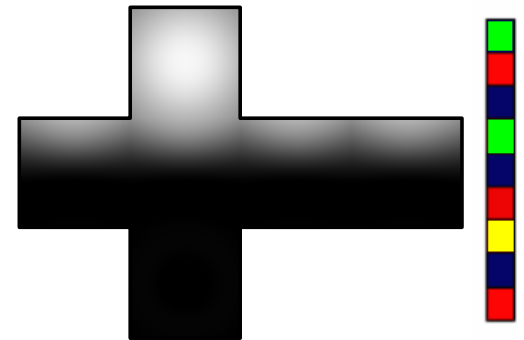


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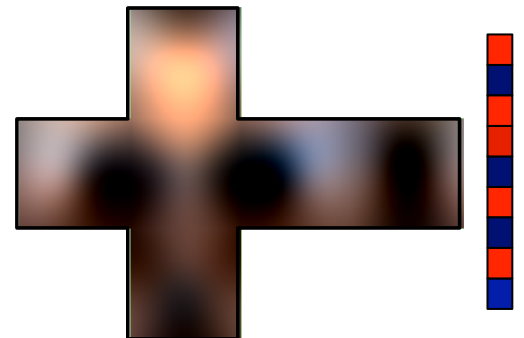
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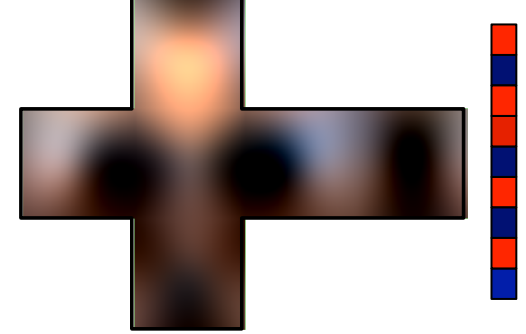


$$\mathbf{L}_e \quad \text{lighting environment}$$



\mathbf{L}_e

lighting environment



- final shading traditionally ([RWS*06;SGNS07]) computed with triple-product SH integration:

$$L_o(\omega_o) = \sum_{ijk} [\mathbf{L}_e]_i [\mathbf{V}]_j [\mathbf{f}_r(\omega_o)]_k \Gamma_{ijk}$$

where

$$\Gamma_{ijk} = \int_{S^2} y_i(\omega) y_j(\omega) y_k(\omega) d\omega$$

are the SH *tripling coefficients*, a sparse order-3 tensor.

- Triple product shading computation is still costly!

Log-BRDF Shading

- We already use log-space to perform a multi-product

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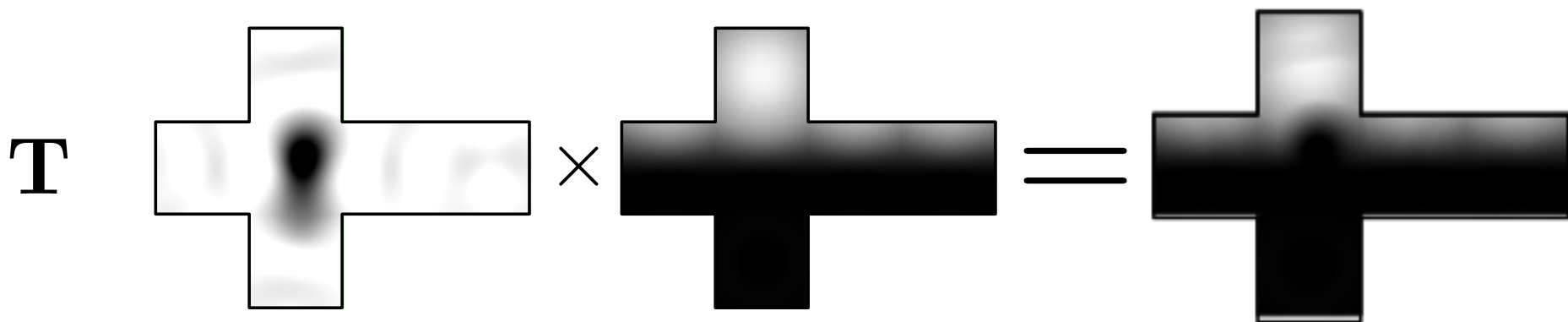
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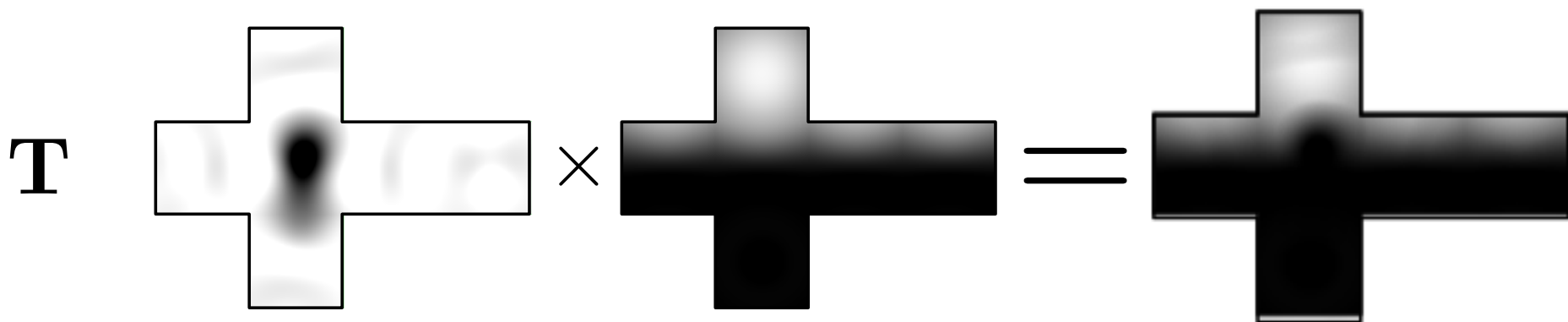
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SH transfer

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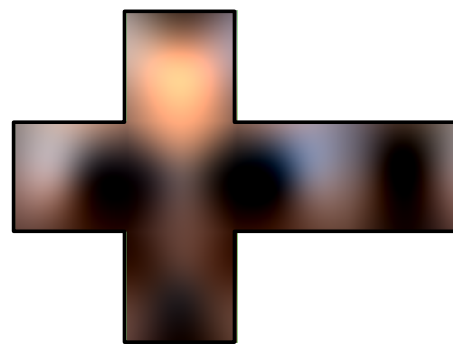
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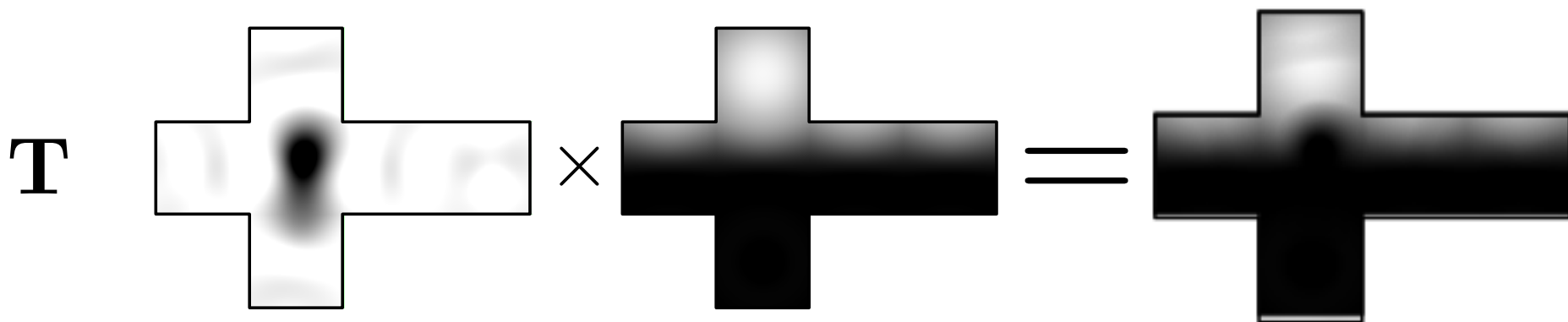
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lighting environment



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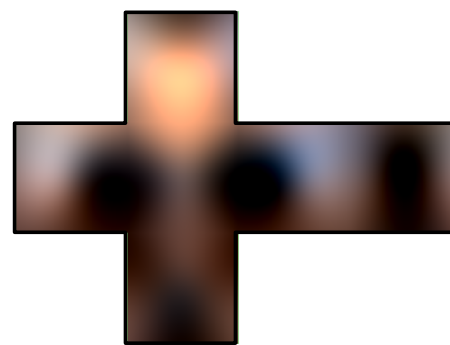
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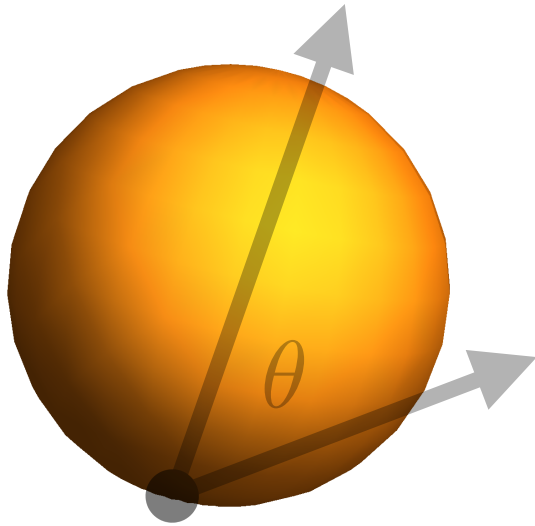
lighting environment



- Now shading requires a cheap *double-product* SH integral!
- but how do we compute the **log-BRDF SH coefficients**?

Log-BRDF SH Coefficients

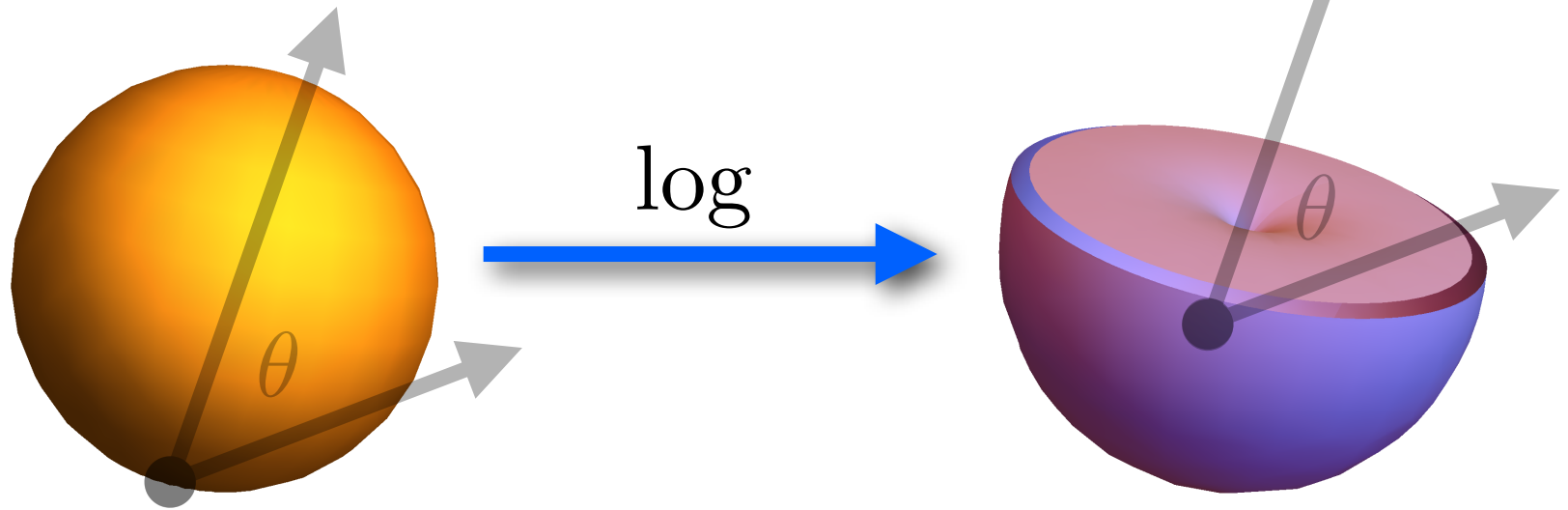
- We compute the log-ZH BRDF coefficients *numerically* for:
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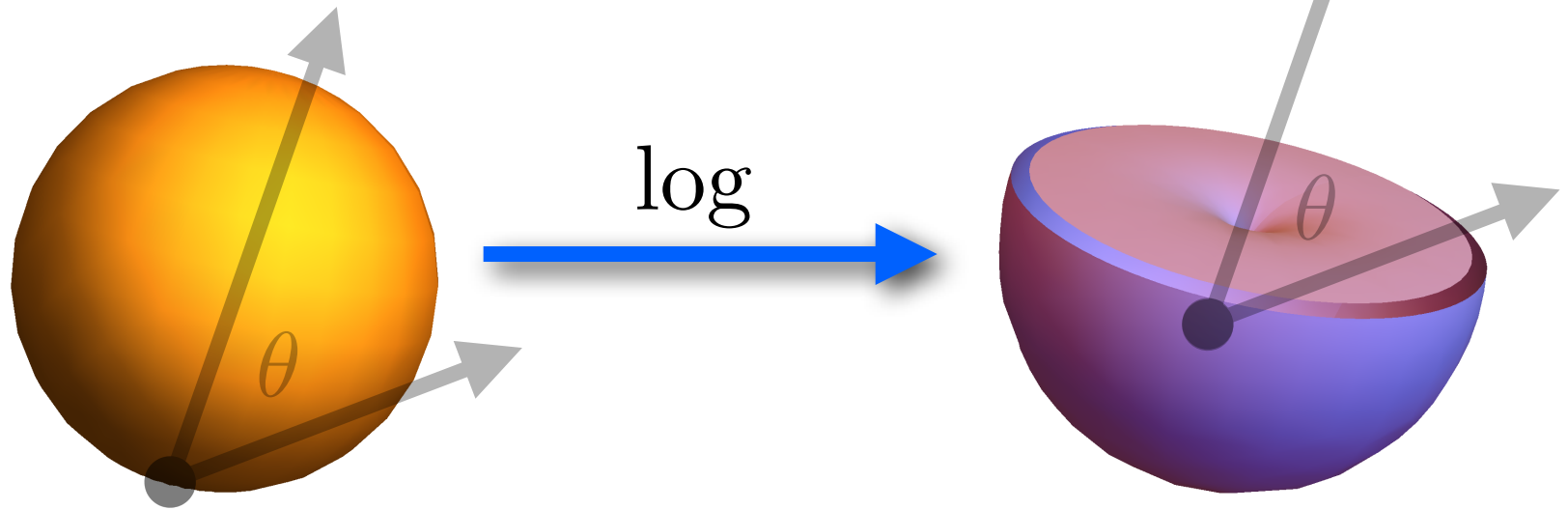
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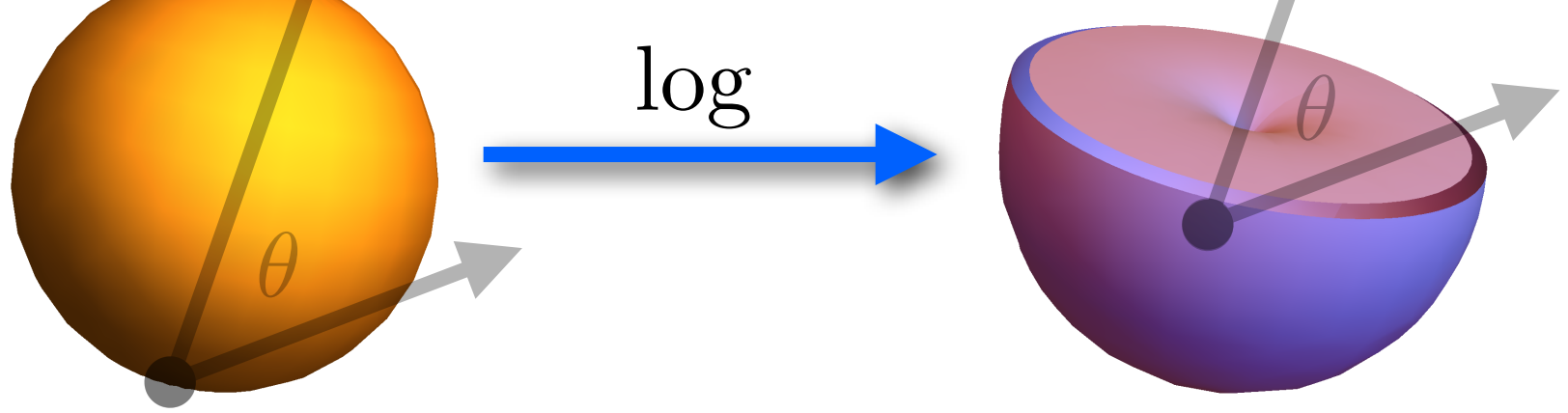
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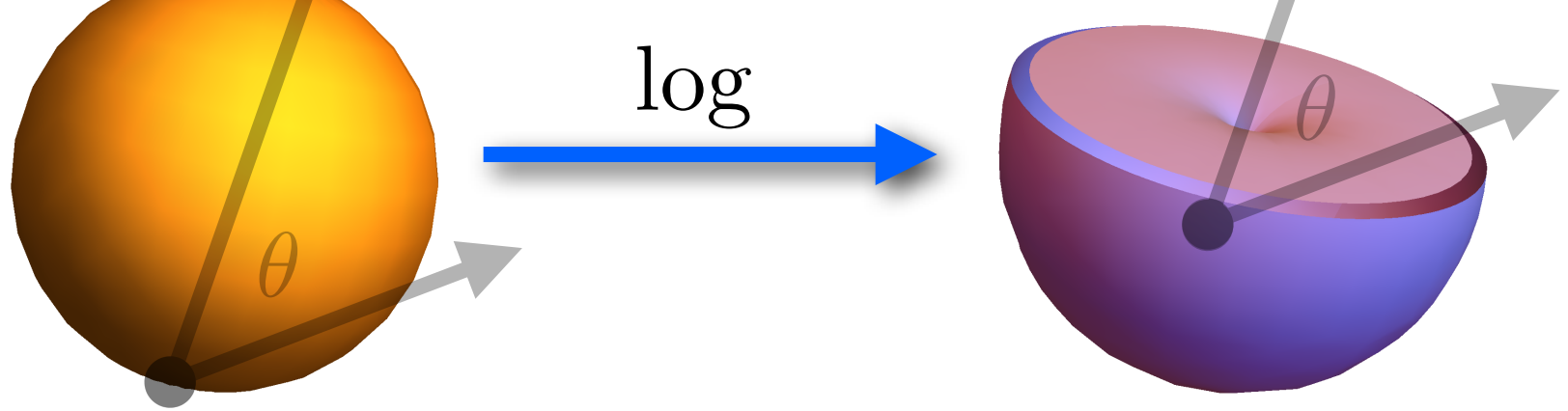
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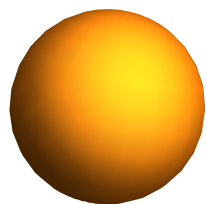
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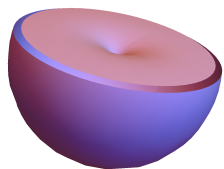
- We compute & tabulate order-4 ZH coefficients numerically

Log-BRDF Error

- In a **worse case** lighting scenario, log-SH BRDF shading still maintains a cosine-like fall-off profile



SH

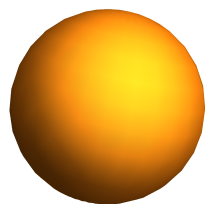


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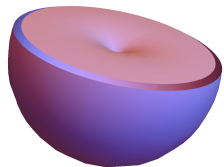
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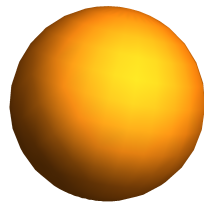
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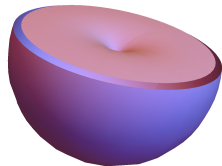
1

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α



1



200



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- Hybrid image/object-space renderer
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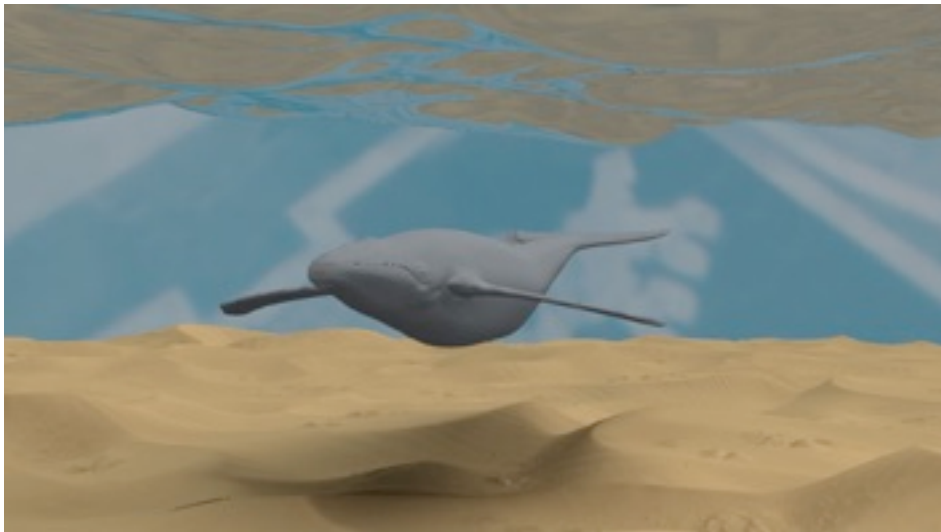
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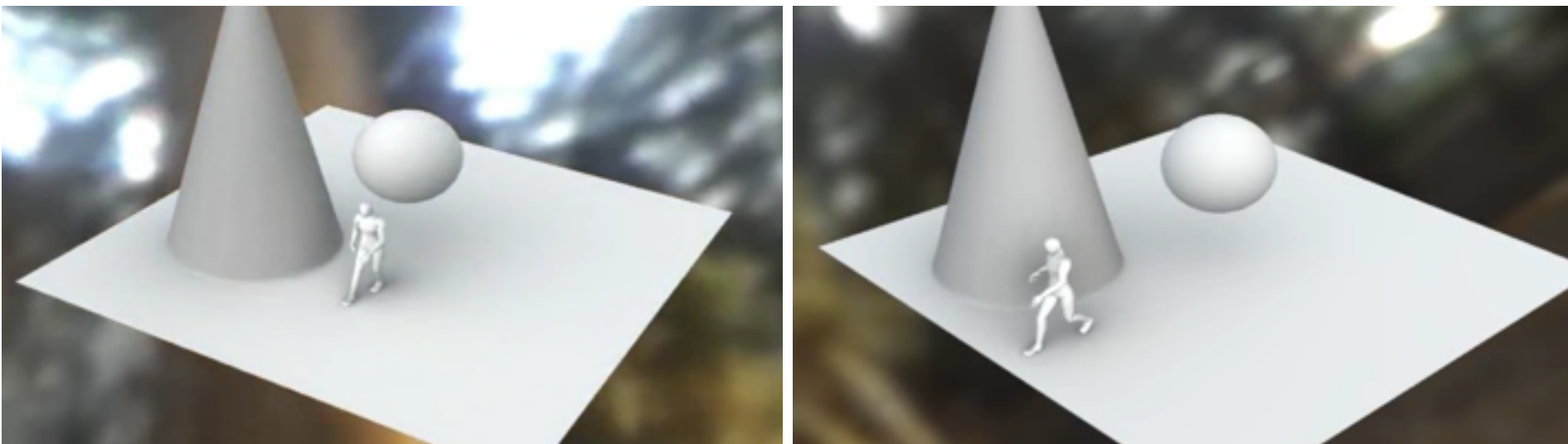
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We acknowledge the helpful suggestions of
the anonymous reviewers.

Thanks! Any questions?